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THESIS

THE EFFECT OF SENSOR PERFORMANCE ON SAFE MINEFIELD TRANSIT

by

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Mines are relatively cheap weapons that can be employed in significant quantity by any country with even a modest military budget, and can be very effective at severely damaging or sinking ships or denying maritime access to an area. In this thesis, simulation and analytical models are formulated and studied to investigate the benefits and risks of mine avoidance, without object classification capability, under circumstances that include imperfect sensors and false targets. Two models of mine avoidance maneuvering are formulated, with increasing complexity in both their analytical and simulation implementations. With both formulations, results are obtained and analyzed to produce tables showing the probability of successful minefield transit as a function of sensor probability of detection vs. density of mine and non-mine, mine-like bottom objects, and the false alarm rate. The tables show the range of those parameter values for which mine avoidance maneuvering improves the probability of safe transit, and the values for which mine avoidance maneuvering reduces the probability of safe transit. The decrease is attributable to the fact that mine avoidance maneuvering increases the distance traveled in the minefield and the consequent risk of damage or destruction by an undetected mine. Quantitative results for the increased distance traveled in the minefield are also presented. Finally, a comparison of the two models of mine avoidance maneuvering show, not surprisingly, that the results of the simpler model are not good approximations of the results obtained with the more complex model, suggesting that even greater complexity in maneuver modeling may be desirable for some purposes.

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THE EFFECT OF SENSOR PERFORMANCE ON SAFE MINEFIELD TRANSIT

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ABSTRACT

Mines are relatively cheap weapons that can be employed in significant quantity by any country with even a modest military budget, and can be very effective at severely damaging or sinking ships or denying maritime access to an area. In this thesis, simulation and analytical models are formulated and studied to investigate the benefits and risks of mine avoidance, without object classification capability, under circumstances that include imperfect sensors and false targets. Two models of mine avoidance maneuvering are formulated, with increasing complexity in both their analytical and simulation implementations. With both formulations, results are obtained and analyzed to produce tables showing the probability of successful minefield transit as a function of sensor probability of detection vs. density of mine and non-mine, mine-like bottom objects, and the false alarm rate. The tables show the range of those parameter values for which mine avoidance maneuvering improves the probability of safe transit, and the values for which mine avoidance maneuvering reduces the probability of safe transit. The decrease is attributable to the fact that mine avoidance maneuvering increases the distance traveled in the minefield and the consequent risk of damage or destruction by an undetected mine. Quantitative results for the increased distance traveled in the minefield are also presented. Finally, a comparison of the two models of mine avoidance maneuvering show, not surprisingly, that the results of the simpler model are not good approximations of the results obtained with the more complex model, suggesting that even greater complexity in maneuver modeling may be desirable for some purposes.

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EXECUTIVE SUMMARY

A classical mine is a weapon that cannot move and can only attack a target by self-destructing. This is a rather primitive approach to warfare. Being required neither to move nor to project power at a distance, mines are relatively cheap. A mine may cost thousands of dollars while a missile or torpedo of equivalent destructive power would cost hundreds of thousands of dollars. Mines can be employed in significant quantity by any country with even a modest military budget, as they are cheap and available on the international arms market. They can be very effective. As seen from the above sentences, a mine is a serious weapon in naval warfare. A minefield can destroy ships and delay access to an area.

In this thesis, simulation and analytical models are formulated and studied to investigate the benefits and risks of mine avoidance under circumstances that include imperfect sensors and false targets. False targets can be non-mine, mine-like bottom objects (NOMBOs), or false alarms generated by random noise in the sensor's receiver. Two models of mine avoidance maneuvering are formulated, with increasing complexity in both their analytical and simulation implementations. One is the Simple Minefield Transit (SMT) Model, and the other is the Minefield Object Avoidance Maneuver (MOAM) Model.

The minefield is considered a rectangle that is, for all practical purposes, infinitely wide; i.e., a ship cannot simply go around the minefield – it must cross it to accomplish its mission. The distance across the minefield is a fixed, finite distance, L. The positions of mines and NOMBOs in the minefield are modeled as independent, homogeneous spatial Poisson processes (Ross, 2000), with intensity parameters that represent the average number of mines or NOMBOs, respectively, per unit area in the minefield.

In the Simple Minefield Transit (SMT) model, when the ship encounters a mine or NOMBO, or the sensor gives a false alarm, the ship retraces its route back to the entry

to the minefield, moves to a different location and attempts to cross the field again along a straight path that does not intersect any of its previous attempts

In the MOAM model, when the ship encounters a detected mine or NOMBO, or the sensor gives a false alarm, the ship attempts to go around the location of the detected object. The ship goes an avoiding distance to the right (for illustration, could alternatively go to the left). If the ship does not detect a mine or NOMBO, or the sensor does not give a false alarm and the ship survives the distance, then it once again proceeds across the field. If the ship encounters a mine or NOMBO, or the sensor gives a false alarm while going to the right, the ship backtracks and tries an avoiding distance to the left; if it does not detect an object and the sensor does not give a false alarm and survives during this avoidance path, it once again proceeds across the field. If the ship encounters a detected mine or NOMBO, or the sensor gives a false alarm in both directions and the ship survives, the ship goes back to the entry to the field, moves to a different location and starts over again.

The simulations are used for two purposes in this thesis. First, the results for probability of safe transit are compared with the analytical model results for verification of both formulations. In addition, the simulation enhances the analytical results by providing additional information such as the distribution of the distance traveled in the minefield and counts of path retracing.

The simulations are written in the JAVA programming language, and are run by typing the appropriate input parameters in the command window. The simulations consider three special cases determined by the input parameters. The first case is a mine only case. This case is simulated when the rate of occurrence of NOMBOs and the rate of false alarms are zero. The second special case is a mine and NOMBO case. This is simulated when the rate of false alarms is set equal to zero. Finally, the third case is a mine, NOMBO, and false alarm case. This is simulated when all three rate parameters are positive. The input parameters include the rate of occurrence of mines, the rate of occurrence of NOMBOs, the rate of false alarms, Y-axis distance of the minefield (L), the mine actuation width of the ship, and the probability of the detection of mines and

NOMBOs by the sensor onboard the ship. In particular, the ROC [1] curve model determines the false alarm rate. This model computes the probability of a false alarm (P_f) based on a given probability of detecting a mine or NOMBO (P_d), and the rate of occurrence of false alarms is calculated by using the P_f. [Appendix A] (Pilnick, 2002).

The output of the simulations include the following: the estimated distribution and mean of the conditional distance traveled given unsuccessful transit; the estimated distribution and mean of the conditional distance traveled given transit is successful; the estimated distribution and mean of the total distance traveled; the mean number of retracings (returns to the entry to the minefield); and the percentage of simulation runs resulting in successful transit, which provides a statistical estimate for the probability of safe transit.

Particular issues studied with both the analytical and simulation models are the probability of safe passage across the minefield and the distance traveled to successfully, or unsuccessfully, transit the field. The histograms of distances traveled suggest that even if the ship successfully transits the minefield, it may need to transit a substantial distance while doing so. The distance traveled is a function of the rate of occurrence of NOMBOs and the rate of occurrence of false alarms. Thus, even if a ship transits the field successfully, it may take a surprisingly long time. The simulation promotes valuable understanding of this situation.

A primary Measure of Effectiveness (MOE) under investigation is the probability of a safe minefield transit, and if, and how, the change of rate of the occurrence of NOMBOs $\left(l_o\right)$ or detection index (d) affects this probability.

The results of this study demonstrate that if NOMBOs exist in the minefield, the probability of a safe minefield transit does not always increase with increasing sensor probability of detection, but sometimes decreases. That is, since detected NOMBOs and false alarms cause the ship to travel greater distances within the field, it is possible for use of the sensor to decrease the probability of successful transit of the minefield. However, if the probability of detection is high enough, then the advantage of being able

[1] ROC: Receiver Operating Characteristic.

to detect an encountered mine outweighs the disadvantage of a longer distance traveled. When there are no NOMBOs in the field, and the probability of detection increases, even if false alarms occur and the rate of occurrence of false alarms is great, the probability of a safe minefield transit always increases. In other words, if it is possible to guarantee that no NOMBOs exist in the minefield, sensors must be used to transit the minefield, even though the detection index is low, because the probability of a safe minefield transit with a sensor is always greater than that with no sensor. However, in the real world, this situation seldom occurs. Thus, how can the ship transit the minefield safely? First, the rate of occurrence of unknown NOMBOs in the minefield should be reduced. The rate of occurrence of mines is not controllable since enemy forces deploy mines. However, surveying the bottom continuously during peacetime and keeping data about the locations of objects on the bottom can reduce the rate of occurrence of unidentified NOMBOs. Next, reducing the rate of false alarms can be accomplished by improving the sensor signal-to-noise ratio.

In this study, the capability to classify an object that is detected, even with some error, is not considered. Thus, when the ship detects something in the minefield, it must attempt to avoid the detected object without classification. However, the results of the mine only case and the mine plus false alarm case can be used to study the advantage of having a perfect classification capability for mines and NOMBOs.

Finally, a comparison of the two models of mine-avoidance maneuvering shows that the results of the simpler SMT model do not agree with the results obtained with the more complex MOAM model, suggesting that even greater complexity in maneuver modeling may be desirable for some purposes. The results of the simpler model are more pessimistic, which is not surprising.

Successful use of the mine avoidance tactic without a sensor that can accurately distinguish between mines and NOMBOs may be limited to those situations for which the rate of occurrence of NOMBOs and false alarm rates are small. Since similar conclusions are obtained from both models, the results suggest that these conclusions usually apply and are not artifacts of the model representation of avoidance maneuvering.

I. INTRODUCTION

A. BACKGROUND

> When a $1,000 mine can damage so severely a $1,000,000,000 ship … it is time to do something about it.
>
> Admiral Edney, 1991

A classical mine is a weapon that cannot move and can only attack a target by blowing itself up. This is a rather primitive approach to warfare. Being required neither to move nor to project power at a distance, mines are relatively cheap. A mine may cost thousands of dollars while a missile or torpedo of equivalent destructive power would cost hundreds of thousands of dollars. Mines can be employed in significant quantity by any country with even a modest military budget, as they are cheap and available on the international arms market. They can be very effective.

During the Korean War, North Korea, with no real navy of its own, was able to mine its harbors and coasts with impunity. The Soviets provided devices and expertise, and the North Koreans used simple junks and sampans to deploy thousands of deadly explosives, often at night, over hundreds of square miles. According to Arnold S. Lott in the "Most Dangerous Sea" (1959), Soviet personnel not only trained North Koreans and supervised mine assembly, but actually laid magnetic mines off Korean coasts.

Although the primary purpose of North Korea's mines was to obstruct U.S. troop and supply movements, plenty of direct damage was also inflicted. On Sept. 26, 1950, the destroyer *Brush* triggered a mine and nine men were killed. Four days later, the destroyer *Mansfield* set off another mine and five more men were killed.

In October 1950, an amphibious task force of 250 ships with around 50,000 troops embarked and steamed back and forth outside the approaches to the Wonsan harbor in North Korea. D-day for the landing at Wonsan had been set for 20 October, but a week after D-day, the task force still marched and countermarched offshore while food supplies ran low. The landing was delayed because the approaches to Wonsan were mined. The minefield in Wonsan harbor inspired RADM Alan Smith to say,

> The US Navy has lost control of the sea to a nation without a Navy, using pre-World War I weapons laid by vessels that were utilized at the time of the birth of Christ. (Melia, 1991)

Admiral Forrest P. Sherman, who was then CNO, later explained,

> … When you can't go where you want to, when you want to, then you haven't got command of the sea. And command of the sea is a rock-bottom foundation for all our war plans. We've been very submarine-conscious and air-conscious. Now we're getting mine-conscious, beginning last week.

That minefield delayed the planned landing at Wonsan by over a week. The United States Navy lost four minesweepers in the process of clearing the mines, and several other ships were also sunk or damaged. (Hartmann, 1979)

During the Gulf War, Iraqi mining operations in the coastal waters and prospective assault beaches directly influenced plans for possible amphibious operations. ADM Arthur (COMUSNAVCENT) said the following about Iraq's use of mines in the Gulf War,

> Iraq successfully delayed and might have prevented an amphibious assault on Kuwait's assailable flank, protected a large part of its force from the effects of naval gunfire, and severely hampered surface operations in the northern Arabian Gulf, all through the use of naval mines. (Mardola and Schneller, 1998)

During Operation Desert Storm, Admiral Frank B. Kelso II said,

> I believe there are some fundamentals about mine warfare we should not forget. Once mines are in place, they are quite difficult to get rid of. That is not likely to change. I think that it is probably going to get worse, because mines are going to get more sophisticated.

It took several months for the allied nations to clear the Iraqi minefields even when their location and nature were revealed after the war.

As seen from the above quotations, a mine is a serious weapon in naval warfare. A minefield can destroy ships and delay access to an area.

The following mine avoidance tactic has been proposed for a ship to cross a minefield. A ship uses a sensor to locate mine-like objects[2] and then maneuvers to avoid the detected objects. It is hoped that this mine avoidance tactic will result in safe transit of the field in a timely manner using fewer resources than either mine sweeping or mine hunting.

In this thesis, models to investigate the benefits/risks of mine avoidance which includes imperfect sensors and false targets are formulated and studied. Particular issues studied with the models include the probability of safe passage through the minefield and the distance traveled to successfully and unsuccessfully pass through the field. The results of this thesis provide guidance as to when mine avoidance tactics can be used rather than the more resource intensive and time consuming tactics of mine hunting or mine sweeping.

B. MINEFIELD TRANSIT MODEL

This thesis models the effect of a sensor used by a minefield transiting ship using a mine avoidance tactic without object classification through analytical models and simulation to analyze the results of transiting either with or without using the sensor.

For instance, without sensors, the optimum path for a ship to transit a minefield may be by a straight line at the field's narrowest point. When a sensor is present, the probability of safe transit may be higher or lower than in the no sensor case, depending on how the sensor is employed. The results of this thesis provide guidance as to when mine avoidance tactics can be used rather than the more resource intensive and time consuming tactics of mine hunting or mine sweeping.

The probability of safe transit as a function of the sensor's performance, minefield density, and false target density is assessed, for example, when a sensor with a specified detection probability confronts a field of x mines/mile^2 and y false targets/mile^2,

- What is the probability of safe transit?
- What combinations of parameters increase/decrease the probability of safe passage?

[2] Mine-like objects include both mine and non-mine, mine-like bottom objects.

1. Mine Distribution in a Minefield

An initial model for the positions of mines in a minefield is the spatial Poisson process (Ross, 2000).

Assume mines are distributed in the field according to a homogeneous spatial Poisson Process with rate l, where l is the expected number of mines/unit area.

- The number of mines in disjoint regions are independent random variables
- There is at most one mine for each location. (no stack)

Let *N(A)* be the number of mines in subregion *A*. The probability that *N(A)* equals n is modeled as

$$P\{N(A)=n\}=e^{-m(A)}\frac{[m(A)]^{n}}{n!} \quad \text{for n = 0, 1, …}$$

where

$$m(A)=l\underbrace{|A|}_{\substack{\text{area of}\\ \text{subregion A}}}=\iint_A l\,dxdy \underset{general}{\overset{in}{=}} \iint_A l(x,y)dxdy$$

Assume mines are distributed according to a Poisson process with rate l

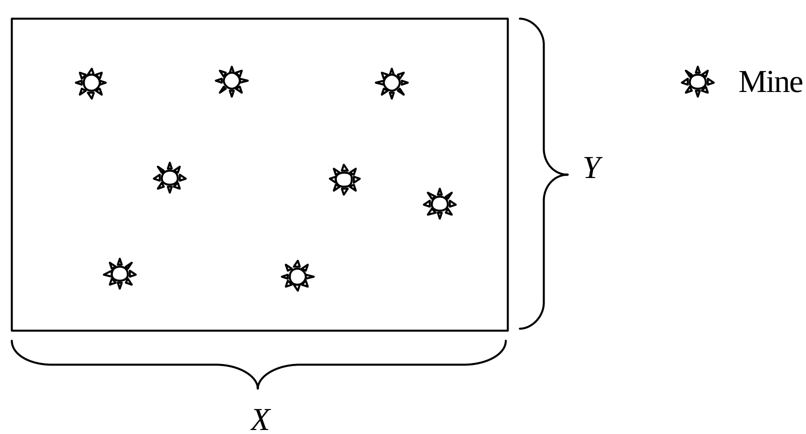

Figure 1. Mine Distribution in a Minefield.

The total area of the field is represented by $a(\bar{R})$. The total region which has area $a(\bar{R})$ is represented by $\bar{R}$. The total number of mines in the region is modeled as a Poisson random variable with mean $l * a(\bar{R})$.

2. Initial Minefield Transit Model

As a preliminary step to modeling imperfect sensors and mine-like objects, an initial model considers a special case of a perfect sensor in order to examine models for alternate paths through the minefield of distance *L* across that take into account a simple diversion tactic that is required as mines are encountered. Assume the ship sees all mines it encounters and no NOMBOs[3] and there are no false alarms. That is, the ship's sensor is perfect. Thus, the ship will successfully pass through the field. However, it may be delayed. Assume the ship has a mine actuation width *w* (*w/2* is the distance between the mine and the center of the ship).

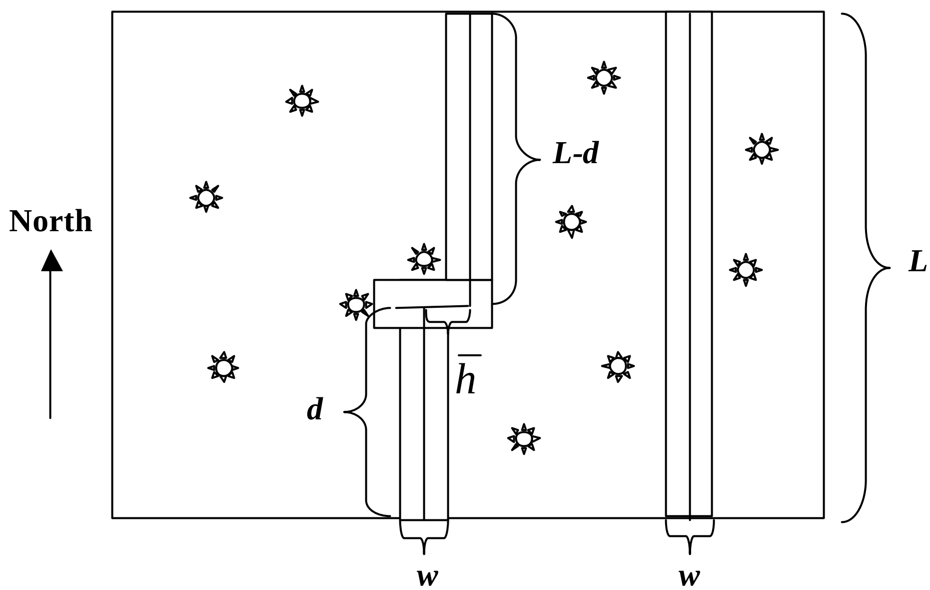

Figure 2. Minefield Transit.

3 NOMBOs: NOn-mine, Mine-like Bottom Objects

What is the probability that the ship can travel more than distance d until it encounters the first mine?

$$P\{D > d\} = P\{N(A) = 0\} = e^{-l|A|}\frac{(l\,|A|)^0}{0!} = e^{-l\,(w*d)} = e^{-(l\,w)d}$$

D = distance until ship encounters the first mine

A = rectangle with length d and width w

$|A|$ = area of A

L = distance of the minefield

As a special case, the probability that the ship does not encounter any mines while transiting the minefield is

$$P\{encounter\ 0\ mines\ while\ transiting\ minefield\} = e^{-(l\,w)L}$$

Now consider that the ship encounters a mine at $L>d$. In this case, the ship will evade the mine by changing course to the left or right and proceed by $\bar{h}$. If no other mines are encountered at the proceeding course, the ship will change course to the north and proceed another distance, L-d, to complete the minefield transit.

When the ship encounters a mine, one evasive action is as follows:

$\bar{h}$ = avoiding distance; distance moved to the left or right to go around perceived mines

h_1, h_2 = actual distance proceeded to the left or right[4]

[4] The avoiding distance and the actual distances proceeded to the left or right during avoidance maneuvers are measured from a point $w/2$ distance units left or right of the original track, respectively. This detail is omitted from the illustrations of the mine avoidance maneuver for clarity.

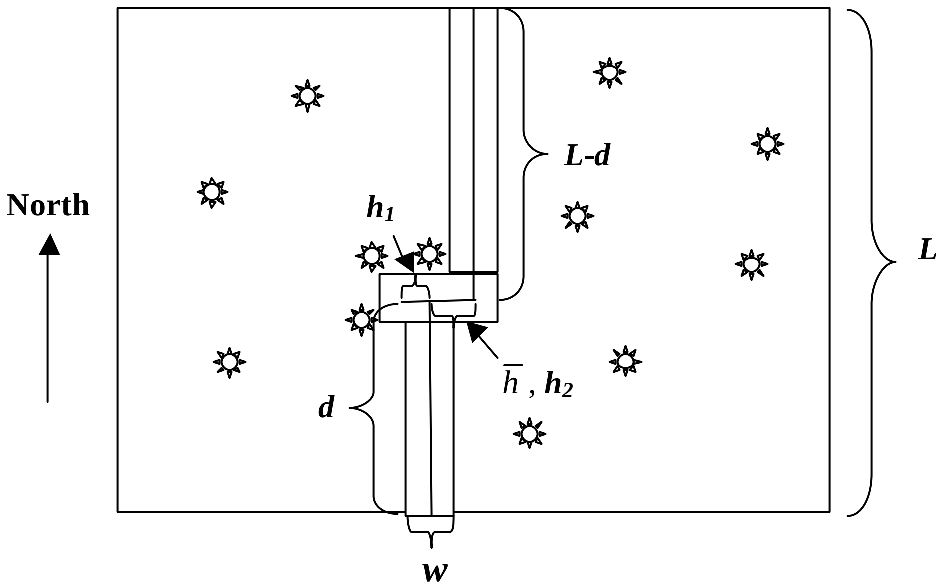

Figure 3. Minefield Transit Case 3.b.(1).

Minefield transit procedure

1. Start minefield transit

2. Encounter no mine ➔ continue north (distance traveled = L)

3. Encounter mine going north across minefield (detection distance ≥ $w/2$) ➔ Turn west (for illustration, could alternatively turn east)

a. Encounter no mine by $\bar{h}$ ➔ turn north.

b. Encounter mine at $h_1 < \bar{h}$ ➔ reverse the direction.

(1) Encounter no mine by $h_1 + \bar{h}$ ➔ turn north.

(2) Encounter mine at $h_2 < \bar{h}$ ➔ go back to the start position of the minefield and find another start position.

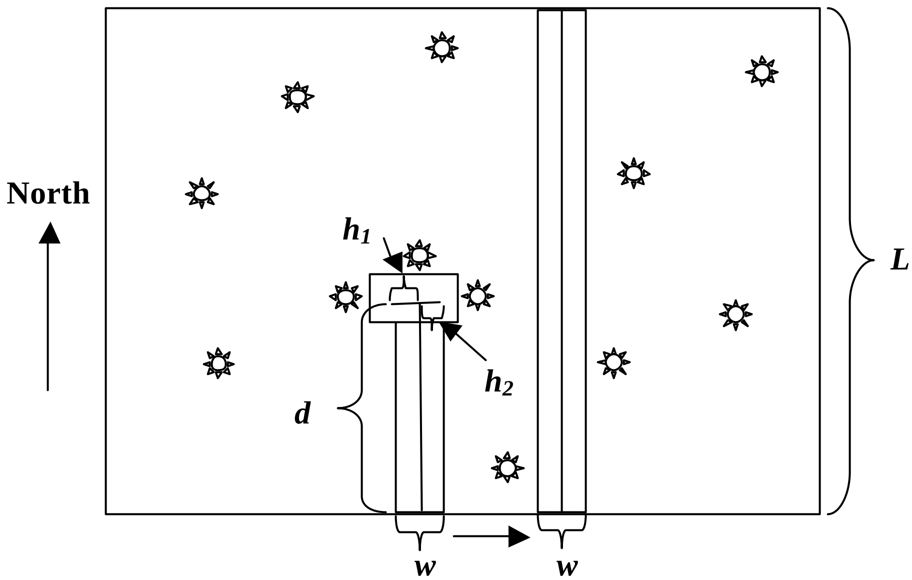

Figure 4. Minefield Transit Case 3.b.(2).

P{successfully evade the mine in the first direction} =

P{no mine in area $w^* \bar{h}$ *}* $= e^{-l w \bar{h}}$

P{successfully evade the mine} = $\underbrace{e^{-l w \bar{h}}}_{\text{1st direction success}} + \underbrace{\left(1 - e^{-l w \bar{h}}\right)}_{\text{1st direction failure}} \underbrace{e^{-l w \bar{h}}}_{\text{2nd direction success}}$

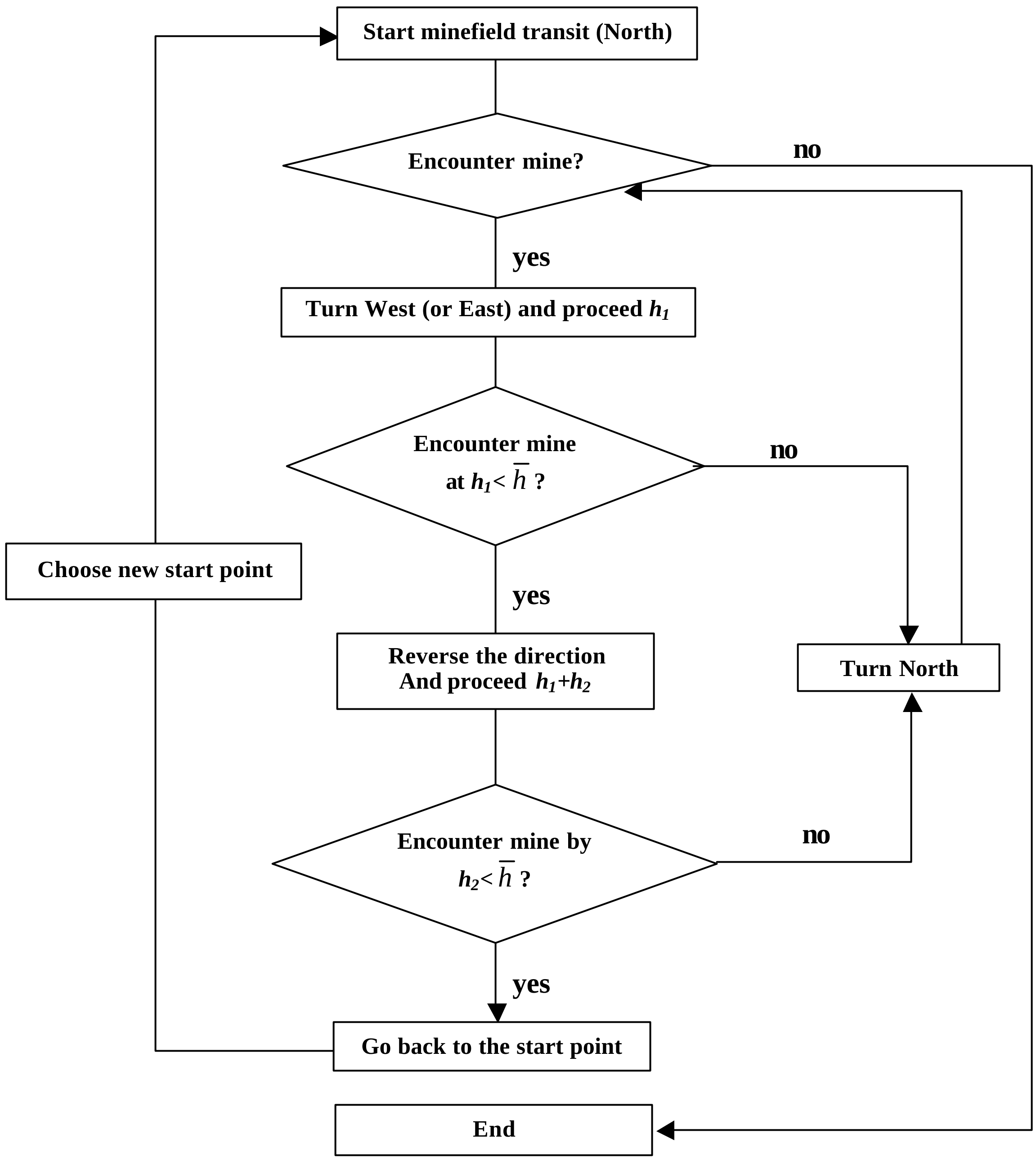

Figure 5. Flow Chart of Minefield Transit with a Perfect Sensor.

This model does not depend on which direction the first avoiding maneuver takes – it could be arbitrarily chosen.

If the evasion is unsuccessful, the ship could return to the start position of the field and start again from another entering place.

In this case, the total traveled distance of the first unsuccessful trial is $2d+2h_1+2h_2$.

Restart on a path that will not overlap the first area searched.

$C_1(d)$ is a successful event after the first avoided direction which occurred a distance d into the field.

P{success through remainder of field | $D_1=d$*, 1st evading direction is success*[5]*} =*

P{no mines in rectangle of area w(L-d)} =* $e^{-\lambda w(L-d)}$

P{success through remainder of field | $D_1=d$*, first evading direction is a failure, second evading direction is a success*[6]*}*

$= e^{-\lambda w(L-d)}$

Models that include an imperfect sensor and false targets also are formulated and studied. Analytical results are obtained for the probability of safe mine field transit. The JAVA programming language and Simkit (Buss, 2002) are used extensively throughout this thesis to evaluate the probability of safe minefield transit using simulation.

Particular issues studied with the models include the probability of safe passage through the minefield, the conditional expected value of the distance the ship transits through the field given it encounters and does not detect a mine, the conditional expected value of the distance the ship travels through the field given it passes through the field safely, and the expected value of number of path retracing. These outputs are used to verify the simulation results with those of the analytical model in Chapters II, III, V, and VI.

The analytical model and simulation model are used to provide recommendations on conditions when a mine avoidance tactic can usefully be employed to transit a minefield.

5 There is no mine in the first evading direction.

6 There is a mine in the first evading direction.

II. SIMPLE MINEFIELD TRANSIT MODEL

A. ANALYTICAL MODEL

Simplified stochastic models (SMT Model; Simple Minefield Transit Model) are presented in this section (Jacobs, 2002).

1. Mine Only Case

Consider a rectangular minefield of distance L across.

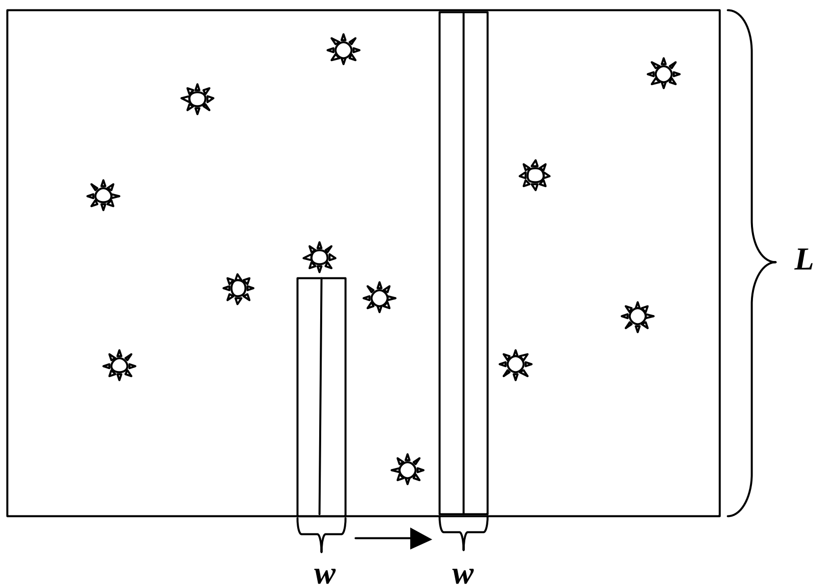

Figure 6. Minefield Transit in SMT Model.

Assume the positions of mines in the region can be well summarized by a Poisson process with rate l_M. A ship is to travel from the entry to the field to the top. The ship has an effective width w. The ship has an imperfect sensor to detect mines. Assume that when a ship encounters a mine, it will detect it with probability $P_d(M)$ independent of the other mines. If the ship encounters a mine without detecting it, the mine will explode and possibly damage the ship. If the ship detects a mine, it retraces its route to the entry to the minefield, moves to a different location along the outside of minefield and attempts to cross the field again along a straight path from the entry to the field to the top that does

not intersect any of its previous attempts. The following is a calculation of the probability that the ship will successfully cross the field.

Let N be the number of mines in the initial path across the field. The effective region induced by the initial path is a rectangle with width w and distance L across and has area wL. The number of mines in this region has a Poisson distribution with mean $l_M * wL$. Let S be the event that the ship successfully crosses the field.

$$P(\ S \mid N=0\ )=1$$
$$P(\ S \mid N>0\ )=\underbrace{P_d(M)}_{\substack{\text{prob first mine}\\ \text{encountered}\\ \text{is detected}}} \quad * \quad \underbrace{P(S)}_{\substack{\text{prob ship successfully}\\ \text{crosses after it returns}\\ \text{to the bottom of the field}}}$$

$$P\{N=0\}=e^{-l_M Lw}, \qquad P\{N>0\}=1-e^{-l_M Lw}$$
$$P(S)=P(\ S \mid N=0\ )P\{N=0\}+P(\ S \mid N>0\ )P\{N>0\}$$
$$=\left[1*e^{-l_M Lw}\right]+P_d(M\ )P(\ S)\left[1-e^{-l_M Lw}\right]$$

This is an equation for P(S). Solving for P(S)

$$P(S)=\frac{e^{-l_M Lw}}{1-\left(1-e^{-l_M Lw}\right)P_d(M)}$$

Note, if $P_d(M)=1$, then $P(S)=1$.

If $P_d(M)=0$, then $P(S)= P\{N=0\}=e^{-l_M Lw}$

2. Mine, NOMBO, and False Alarm Cases

Assume that the field contains not only mines but also NOMBOs and the sensor can give false alarms. If the ship detects a NOMBO or the sensor gives a false alarm, the ship will retrace its route to the entry to the field and attempt to cross the field again along a straight-line path that does not intersect the previous abandoned path(s). Such a path will always exist for a minefield with infinite width. Assume detected NOMBOs occur according to a Poisson process with rate $l_O p_d(O)$ and false alarms occur according to a Poisson process with rate l_F. Let N be the number of events requiring action, e.g., encountering a mine, detecting a NOMBO or experiencing false alarms that occur along the initial path through the field.

Do = distance until there is a NOMBO.

Do has an exponential distribution with mean $\frac{1}{l_O}$.

Do(d) = distance until a NOMBO is detected.

Do(d) has an exponential distribution with mean $\frac{1}{l_O P_d(O)}$.

D_F = distance until there is a false alarm.

D_F has an exponential distribution with mean $\frac{1}{l_F}$.

$$C = \text{type of event} = \begin{cases} M \quad \text{with prob } \dfrac{l_M}{l_M + l_O P_d(O) + l_F}, \; M = \text{Mine} \\ O_d \quad \text{with prob } \dfrac{l_O P_d(O)}{l_M + l_O P_d(O) + l_F}, \; O_d = \text{Detected Object} \\ F \quad \text{with prob } \dfrac{l_F}{l_M + l_O P_d(O) + l_F}, \; F = \text{False Alarm} \end{cases}$$

$P(S \mid C = M, N > 0) = P_d(M)*P(S)$

$P(S \mid C = O_d, N > 0) = P(S)$

$P(S \mid C = F, N > 0) = P(S)$

$P(S) = P(S \mid N = 0)P\{N = 0\} +$

$P(S \mid N > 0, C = M)P\{N > 0, C = M\} +$

$P(S \mid N > 0, C = O_d)P\{N > 0, C = O_d \} +$

$P(S \mid N > 0, C = F)P\{N > 0, C = F\}$

$$
\begin{aligned}
&= 1 * \overbrace{e^{-(l_M + l_O P_d(O) + l_F)Lw}}^{P\{N=0\}} \\
&+ \overbrace{(1 - e^{-(l_M + l_O P_d(O) + l_F)Lw})}^{P\{N>0\}}\{\overbrace{\frac{l_M}{l_M + l_O P_d(O) + l_F}}^{\text{probability first event is encounter of mine}}[P_d(M)P(S) + (1 - P_d(M)) * 0] \\
&\qquad + \underbrace{\frac{l_O P_d(O)}{l_M + l_O P_d(O) + l_F}}_{\text{probability first event is detection of mine like object}} P(S) + \underbrace{\frac{l_F}{l_M + l_O P_d(O) + l_F}}_{\text{probability first event is a false alarm}} P(S)\}
\end{aligned}
$$

This is an equation for P(S). Solving for P(S)

$$
P(S) = \frac{e^{-(l_M + l_O P_d(O) + l_F)Lw}}{1 - \left(1 - e^{-(l_M + l_O P_d(O) + l_F)Lw}\right)\left[\dfrac{l_M P_d(M) + l_O P_d(O) + l_F}{l_M + l_O P_d(O) + l_F}\right]} \qquad (1)
$$

As a special case, when $l_O = l_F = 0$

$$
P(S) = \frac{e^{-l_M Lw}}{1 - \left(1 - e^{-l_M Lw}\right) P_d(M)}
$$

The result agrees with that for the mine only case.

B. SIMULATION

1. Introduction to Simulation

The simulation discussed below was developed to increase the variety of experimentation possibilities with the Analytical Minefield Transit Model. The simulation is written in the JAVA programming language, and is run by typing the appropriate input parameters in the command window shown in Figure 7 below. The simulation considers three special cases determined by the input parameters. The first case is a mine only case. This case is simulated when the rate of occurrence of NOMBOs

and the rate of false alarms are zero. The second special case is a mine and NOMBO case. This is simulated when the rate of false alarms is set equal to zero. Finally, the third case is a mine, NOMBO, and false alarm case. This is simulated when all three rate parameters are positive.

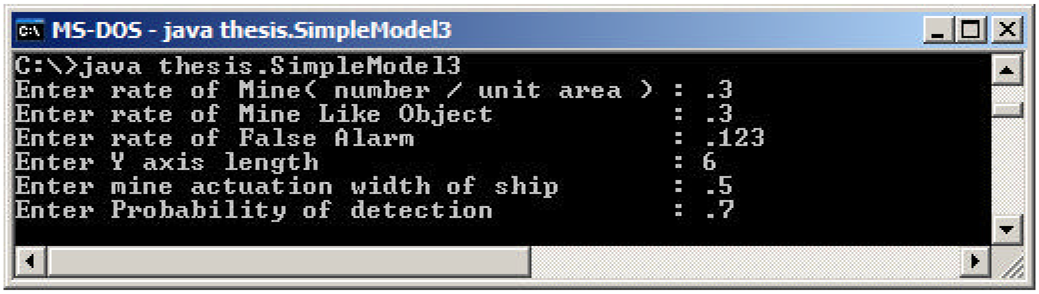

Figure 7. SMT Model Simulation Input Parameters.

Figure 7 shows input parameters. The parameters include the rate of occurrence of mines, the rate of occurrence of NOMBOs, and the rate of occurrence of false alarms, Y-axis length of the minefield, the mine actuation width of the ship, and the probability of the detection of mines and NOMBOs from the sensor onboard the ship. In particular, the ROC[7] curve model determines the false alarm rate. This model computes the probability of a false alarm (P_f) based on a given probability of detecting a mine or NOMBO (P_d) and the rate of occurrence of false alarms is calculated by using the P_f. [Appendix A] (Pilnick, 2002).

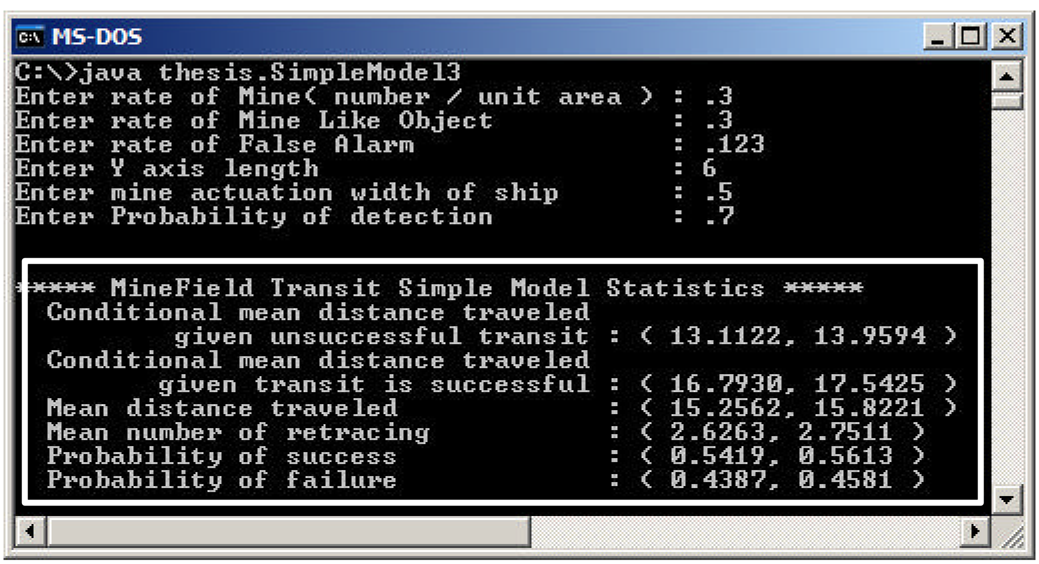

Figure 8. SMT Model Simulation Output.

[7] ROC: Receiver Operating Characteristic.

Figure 8 shows output. The simulation results for probability of success are compared with the analytical model results for verification of both formulations. In addition, the simulation enhances the analytical results by providing additional information shown such as conditional mean distance traveled given unsuccessful transit; the conditional mean distance traveled given transit is successful; the mean distance traveled; and the mean number of retracings (returns to the entry to the minefield). The simulation also provides the distribution of the distance traveled in the minefield and counts of path retracing.

2. The Simulation

Figure 9 below shows pseudo code of a SMT model simulation of a mine only case. The simulation starts with drawing the distance to the first mine (D_M) on the track. If D_M is greater than L, the loop finishes. Else, according to the probability of detecting the mine, the ship will return and enter again at new starting point or be exploded. To obtain output statistics with small standard errors, the simulation is replicated 10,000 times in each run.

```
Distance need is L( Y axis ), Width of mine actuation is w
l_M = E[ # mines / unit area ]
P_d(M) = Probability of detecting mine

Draw distance to first mine is D_M
D_M ~ exp mean 1/( l_M * w)

Do
  If D_M > L
    Finish
  Else
    Uniform(0, 1) ≤ P_d(M)    then return , draw new D, and enter again
    Uniform(0, 1) > P_d(M)    then blow up
Until (Finish or Blow up)
```

Figure 9. Pseudo Code of SMT Model Simulation (Mine Only Case).

Figures 10 and 11 below display pseudo codes of the other SMT model simulations.

```
Distance need is L( Y axis ), Width of mine actuation is w
l_M = E[ # mines / unit area]
l_O = E[ # of NOMBOs / unit area]
P_d(M) = Probability of detecting mine
P_d(O) = Probability of detecting NOMBO

Draw distance to first mine is D_M
Draw distance to first NOMBO is D_O
D_M ~ exp mean 1/( l_M * w)
D_O ~ exp mean 1/( l_O * w)

Do
  If Min(D_M, D_O) > L
    Finish
  Else If D_M ≤ D_O
    Uniform(0, 1) ≤ P_d(M)   then return , draw new D, and enter again
    Uniform(0, 1) > P_d(M)   then blow up
  Else
    Uniform(0, 1) ≤ P_d(O)   then return , draw new D, and enter again
    Uniform(0, 1) > P_d(O)   then draw new D_O ( D_O = D_O + new D_O )
      Do
        If Min(D_M, D_O) > L
          Finish
        Else If D_O ≤ D_M
          Uniform(0, 1) ≤ P_d(O)   then return , draw new D, and enter again
        Else If D_M ≤ D_O
          Uniform(0, 1) ≤ P_d(M)   then return , draw new D, and enter again
          Uniform(0, 1) > P_d(M)   then blow up
      Until (Finish, Return or Blow up)
Until (Finish or Blow up)
```

Figure 10. Pseudo Code of SMT Model Simulation (Mine and NOMBO Case).

```
Distance need is L( Y axis ), Width of mine actuation is w
l_M = E[ # mines / unit area]
l_O = E[ # of NOMBOs / unit area]
l_F = E[ # false Alarms / unit area]

Pd(M) = Probability of detecting mine
Pd(O) = Probability of detecting NOMBO

Draw distance to first mine is D_M
Draw distance to first NOMBO is D_O
Draw distance to first false alarm is D_F

D_M ~ exp mean 1/( l_M * w)
D_O ~ exp mean 1/( l_O * w)
D_F ~ exp mean 1/( l_F * w)

Do
    If Min(D_M, D_O, D_F) > L
        Finish
    Else If D_M ≤ Min(D_O, D_F)
        Uniform(0, 1) ≤ Pd(M)   then return , draw new D, and enter again
        Uniform(0, 1) > Pd(M)   then blow up
     Else If D_F ≤ Min(D_M, D_O)
        then return , draw new D, and enter again
     Else If D_O ≤ Min(D_M, D_F)
        Uniform(0, 1) ≤ Pd(O)   then return , draw new D, and enter again
        Uniform(0, 1) > Pd(O)   then draw new D_O ( D_O = D_O + new D_O )
        Do
            If Min(D_M, D_O, D_F) > L
                Finish
            Else If D_O ≤ Min(D_M, D_F)
                Uniform(0, 1) ≤ Pd(O)   then return , draw new D, and enter again
            Else If D_M ≤ Min(D_O, D_F)
                Uniform(0, 1) ≤ Pd(M)   then return , draw new D, and enter again
                Uniform(0, 1) > Pd(M)   then blow up
            Else If D_F ≤ Min(D_M, D_O)
                then return , draw new D, and enter again
        Until (Finish, Return or Blow up)
Until (Finish or Blow up)
```

Figure 11. Pseudo Code of SMT Model Simulation (Mine, NOMBO Case, and False Alarm Case).

III. INITIAL ANALYSIS OF THE SIMULATION FOR THE SIMPLE MINEFIELD TRANSIT MODEL

A. INTRODUCTION

This chapter compares the simulation results with a numerical example using the analytical SMT model. The fraction of replications resulting in successful minefield transit is compared to the analytical probability of safe minefield transit. This is computed for the mine-only case, mine + NOMBO case, and mine + NOMBO + false alarm case respectively. The probabilities of a false alarm used in the models appear in Table 48 in Appendix A. All simulation runs have 10,000 replications.

B. PROBABILITY OF SAFE MINEFIELD TRANSIT

The tables below show the output of the analytical SMT model.

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
Pd(M)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x	0.165	0.165	0.165	0.165	0.165	0.165	0.165	0.165	0.165	0.165	0.165
P(S)	**0.165**	**0.180**	**0.198**	**0.221**	**0.248**	**0.284**	**0.331**	**0.398**	**0.498**	**0.664**	**1.000**
P(F)	0.835	0.820	0.802	0.779	0.752	0.716	0.669	0.602	0.502	0.336	0.000

*x=exp(-lambdaM*L*w)*

Table 1. Probability of Safe Minefield Transit (Mine Only Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaO	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
Pd(M)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Pd(O)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x	0.165	0.138	0.115	0.096	0.080	0.067	0.056	0.047	0.039	0.033	0.027
P(S)	**0.165**	**0.164**	**0.164**	**0.165**	**0.170**	**0.178**	**0.192**	**0.218**	**0.268**	**0.391**	**1.000**
P(F)	0.835	0.836	0.836	0.835	0.830	0.822	0.808	0.782	0.732	0.609	0.000

*x=exp(-(lambdaM+lambdaO*Pd(O))*L*w)*

Table 2. Probability of Safe Minefield Transit (Mine and NOMBO Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaO	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaF	0	0.002	0.005	0.010	0.017	0.029	0.043	0.062	0.091	0.152	1.101
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
Pd(M)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Pd(O)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
x	0.165	0.137	0.112	0.090	0.073	0.057	0.043	0.032	0.023	0.013	0.000
P(S)	**0.165**	**0.163**	**0.161**	**0.159**	**0.160**	**0.161**	**0.165**	**0.175**	**0.196**	**0.243**	**1.000**
P(F)	0.835	0.837	0.839	0.841	0.840	0.839	0.835	0.825	0.804	0.757	0.000

*x=exp(-(lambdaM+lambdaO*Pd(O)+lambdaF)*L*w)*

Table 3. Probability of Safe Minefield Transit (Mine, NOMBO, and False Alarm Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
Pd(M)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P(S)	**0.165**	**0.198**	**0.237**	**0.284**	**0.340**	**0.407**	**0.487**	**0.583**	**0.698**	**0.835**	**1.000**

*P(S)=exp(-lambdaM*L*w(1-Pd(M)))*

Table 4. Probability of Safe Minefield Transit (Optimistic Case).

The rate of occurrence of mines and NOMBOs used in this analysis is 0.3 mines/mile^2. The minefield distance is 6 miles and the width of the mine actuation is 1 mile. As mentioned previously, the ROC curve model determines the rate of false alarm in Table 3 [Appendix A]. The probabilities of mine detection and NOMBO detection are assumed equal, i.e., $P_d(M) = P_d(O) = P_d$, because, when the ship detects some object in the water, the ship evades the object without classification.

To compare these outputs, an optimistic case is calculated. The optimistic case uses all the assumptions of the analytical model with the exception that, when the ship detects a mine, it will proceed towards the end of the field without diversion and without exploding the mine.

To obtain an upper bound on the probability of safe minefield transit, let

N_u = number of mines undetected in path $L *w$. N_u has a Poisson distribution with mean $l_M\left(L*w\right)\left(1-P_d\left(M\right)\right)$.

An upper bound on the probability of a safe minefield transit is:

$$P(S) = P(N_u = 0) = e^{-l_M(Lw)(1-P_d(M))} \qquad (2)$$

In any case, the probabilities of safe minefield transit will not exceed the optimistic case. Table 4 above shows the upper bounds on the probabilities of a safe minefield transit for the parameter used.

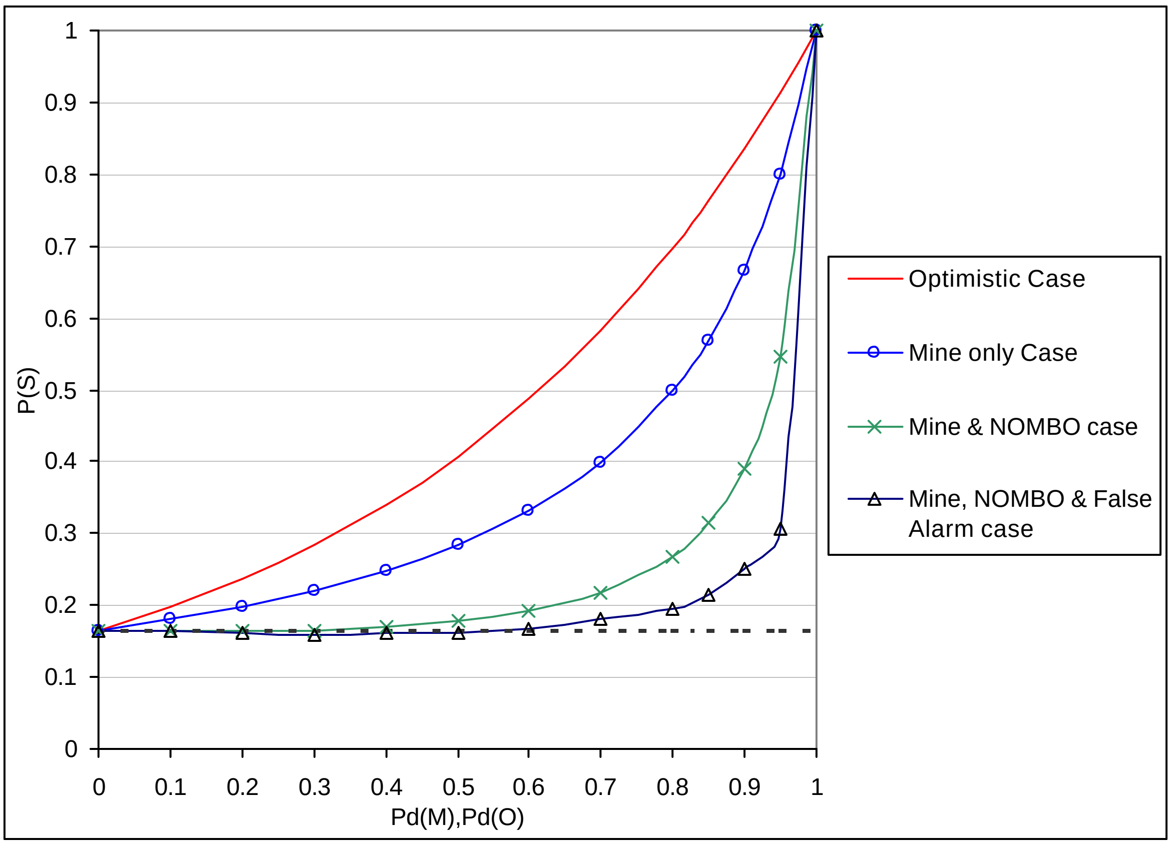

Figure 12. Probability of Safe Minefield Transit (Analytical Model).

Figure 12 shows four cases of the probability of a safe minefield transit. As can be seen in the above graph, the probability of a safe minefield transit decreases when NOMBOs exist in the minefield and false alarms occur, and increases when the probability of detection increases.

Table 5 below shows the estimates of the probabilities that the ship transits the minefield safely for the simulation. Input parameters of simulation are the same as those of the analytical model. The number of simulation replications is 10,000 for each case. The confidence limits are obtained using a normal approximation. (Devore, 2000)

Pd		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
Mine only	+.95 CI	0.172	0.188	0.203	0.224	0.251	0.291	0.339	0.404	0.506	0.677	1.000
	- .95 CI	0.157	0.173	0.188	0.208	0.234	0.273	0.320	0.385	0.487	0.659	1.000
	Mean	**0.165**	**0.181**	**0.196**	**0.216**	**0.243**	**0.282**	**0.330**	**0.395**	**0.497**	**0.668**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.000
Analytical Mean		**0.165**	**0.180**	**0.198**	**0.221**	**0.248**	**0.284**	**0.331**	**0.398**	**0.498**	**0.664**	**1.000**
Mine & NOMBO	+.95 CI	0.172	0.166	0.167	0.170	0.178	0.183	0.201	0.224	0.282	0.404	1.000
	- .95 CI	0.157	0.151	0.152	0.156	0.164	0.168	0.185	0.208	0.265	0.385	1.000
	Mean	**0.165**	**0.159**	**0.160**	**0.163**	**0.171**	**0.175**	**0.193**	**0.216**	**0.273**	**0.394**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.000
Analytical Mean		**0.165**	**0.164**	**0.164**	**0.165**	**0.170**	**0.178**	**0.192**	**0.218**	**0.268**	**0.391**	**1.000**
Mine, NOMBO, & false alarm	+.95 CI	0.172	0.166	0.165	0.161	0.163	0.165	0.170	0.187	0.206	0.257	1.000
	- .95 CI	0.157	0.151	0.151	0.147	0.148	0.151	0.155	0.172	0.190	0.240	1.000
	Mean	**0.165**	**0.158**	**0.158**	**0.154**	**0.156**	**0.158**	**0.163**	**0.180**	**0.198**	**0.248**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.004	0.000
Analytical Mean		**0.165**	**0.163**	**0.161**	**0.159**	**0.160**	**0.161**	**0.165**	**0.175**	**0.196**	**0.243**	**1.000**

Table 5. Estimate of Probability of Safe Minefield Transit (Simulation).

Figure 13 below shows the mean probability of a safe minefield transit and 95% CI graphically. For all cases, the analytical probabilities are within the 95% confidence intervals obtained from the simulation model.

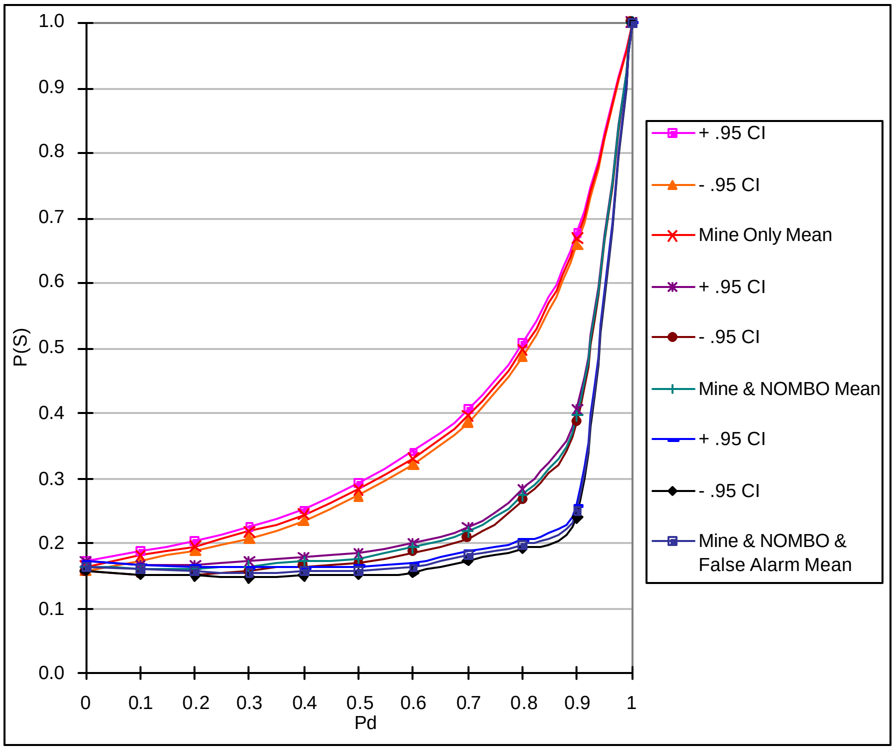

Figure 13. Estimate of Probability of Safe Minefield Transit and 95% CI.

C. OTHER SIMULATION RESULTS

1. Conditional Mean Distance Traveled Given Unsuccessful Transit

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.999
Mine only	+.95 CI	2.1995	2.6371	3.1249	3.6646	4.4201	5.4613	6.7591	8.4912	11.357	16.011	35.668
	- .95 CI	2.1304	2.5333	2.9895	3.4982	4.2112	5.1931	6.4118	8.0368	10.694	14.888	22.271
	Mean	2.1650	2.5852	3.0572	3.5814	4.3156	5.3272	6.5855	8.2640	11.025	15.449	28.969
	Std Err	0.0002	0.0003	0.0004	0.0005	0.0006	0.0008	0.0011	0.0015	0.0024	0.0050	0.461
Mine & NOMBO	+.95 CI	2.1995	2.9438	3.7403	4.7786	6.1102	7.8561	10.396	14.288	21.002	36.625	105.16
	- .95 CI	2.1304	2.8187	3.5707	4.5538	5.8232	7.4812	9.916	13.635	19.999	34.764	78.92
	Mean	2.1650	2.8813	3.6555	4.6662	5.9667	7.6687	10.156	13.961	20.501	35.694	92.04
	Std Err	0.0002	0.0003	0.0005	0.0006	0.0008	0.0011	0.0014	0.0019	0.0030	0.0061	0.500
Mine, NOMBO, & false alarm	+.95 CI	2.1995	2.9991	3.8363	4.9053	6.3854	8.2653	10.928	15.312	24.466	48.004	2125.7
	- .95 CI	2.1304	2.8709	3.6619	4.6790	6.0884	7.8818	10.430	14.627	23.364	45.834	1983.0
	Mean	2.1650	2.9350	3.7491	4.7921	6.2369	8.0735	10.679	14.969	23.915	46.919	2054.4
	Std Err	0.0002	0.0004	0.0005	0.0006	0.0008	0.0011	0.0014	0.0019	0.0031	0.0064	0.656

Table 6. Conditional Mean Distance Traveled Given Unsuccessful Transit (Simulation).

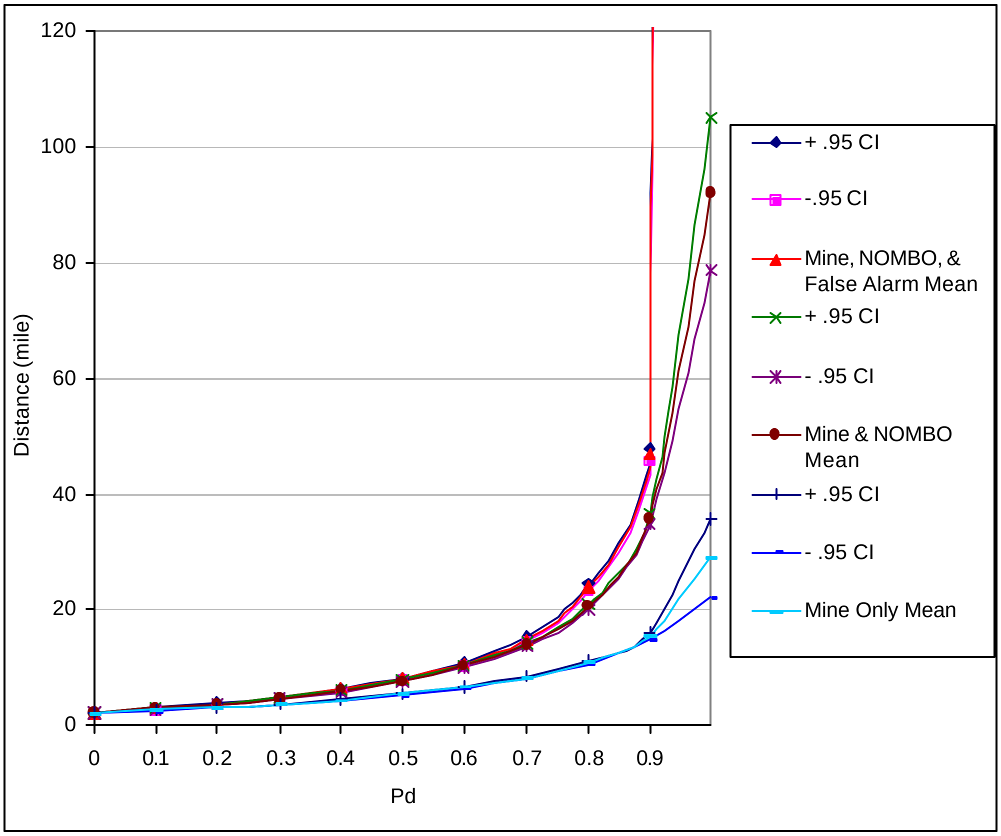

Figure 14. Conditional Mean Distance Traveled Given Unsuccessful Transit and 95% CI.

Table 6 and Figure 14 above show the conditional mean distance traveled given an unsuccessful minefield transit as the probability of detection increases. In this case, the results do not exist when the probability of detection is 1.0. Therefore, the probability of detection 0.999 is used instead of 1.0 to obtain the extreme results. The conditional mean distance traveled given unsuccessful minefield transit increases as the probability of detection increases and it increases quickly when NOMBOs and false alarms exist.

Figures 15 and 16 display histograms of the distance traveled given unsuccessful transit, when there exist mines and NOMBOs in the field and there are no false alarms. Two cases (low and high rates of occurrence of NOMBOs in the minefield) are compared to study how the NOMBO acts on the minefield travel distance given unsuccessful transit. All the assumptions are same as those used earlier in this chapter except that the width of mine actuation is 0.5 miles. The low (respectively high) rate of occurrence of the NOMBOs is 0.3 (respectively 1.5).

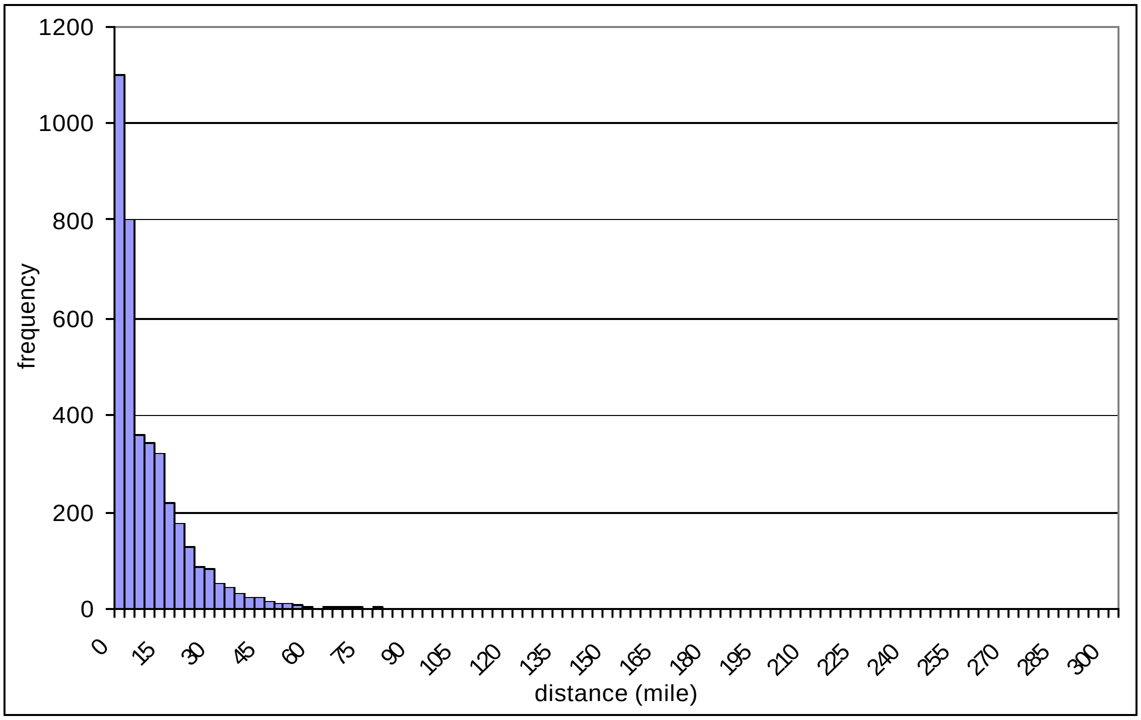

Figure 15. Histogram of Distance Traveled Given Unsuccessful Transit, L=6, w=0.5, l_M =0.3, l_O =0.3, l_F =0.0, P_d=0.7.

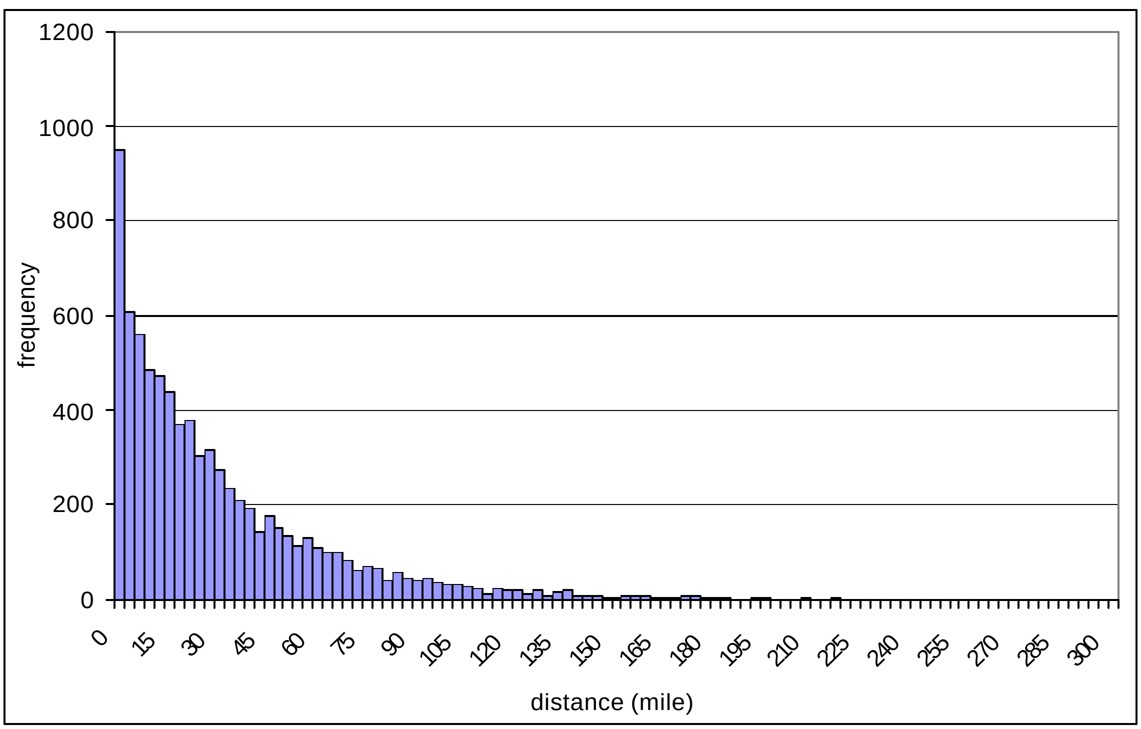

Figure 16. Histogram of Distance Traveled Given Unsuccessful Transit, L=6, w=0.5, l_M =0.3, l_O =1.5, l_F =0.0, P_d=0.7.

The total number of observations displayed in Figure 15 is 3870 and that of Figure 16 is 7913. The mean and maximum distances traveled displayed in Figure 15 are 10.51 miles and 102.56 miles, and in Figure 16 are 32.15 miles and 300.97 miles respectively. This shows us that, when the rate of occurrence of NOMBOs increases in the minefield, the mean and maximum distances traveled before encountering an undetected mine also increase.

2. Conditional Mean Distance Traveled Given Successful Transit

Table 7 and Figure 17 below show the conditional mean distance traveled given a successful transit, as the probability of detection increases. The mean distance traveled given a successful transit increases as the probability of detection increases. According to the ROC curve model, when the probability of detecting mine ($P_d(M)$) is 1, the probability of a false alarm (P_f) is almost 1. As a result, the rate of a false alarm (l_F) becomes 1.101, which makes the total distance extremely long.

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mine only	+.95 CI	6.0000	6.5186	6.9953	7.5699	8.3665	9.2197	10.512	12.260	14.963	19.670	28.452
	- .95 CI	6.0000	6.3494	6.7609	7.2813	8.0186	8.8310	10.036	11.711	14.308	18.884	27.465
	Mean	6.0000	6.4340	6.8781	7.4256	8.1926	9.0253	10.274	11.986	14.636	19.277	27.958
	Std Err	0.0000	0.0010	0.0014	0.0016	0.0018	0.0019	0.0021	0.0022	0.0024	0.0025	0.0025
Mine & NOMBO	+.95 CI	6.0000	6.9054	7.8685	9.0282	10.478	12.522	14.876	18.765	25.818	41.059	115.70
	- .95 CI	6.0000	6.6714	7.5050	8.5388	9.848	11.693	13.907	17.536	24.179	38.707	111.25
	Mean	6.0000	6.7884	7.6868	8.7835	10.163	12.107	14.392	18.150	24.998	39.883	113.48
	Std Err	0.0000	0.0015	0.0023	0.0031	0.0039	0.0050	0.0056	0.0067	0.0080	0.0096	0.0113
Mine, NOMBO, & false alarm	+.95 CI	6.0000	6.8358	7.9987	9.2143	10.385	13.124	16.087	20.364	30.127	53.176	32488
	- .95 CI	6.0000	6.6162	7.6071	8.6904	9.743	12.246	14.918	18.811	27.855	49.420	31240
	Mean	6.0000	6.7260	7.8029	8.9523	10.064	12.685	15.502	19.587	28.993	51.298	31864
	Std Err	0.0000	0.0014	0.0025	0.0034	0.0042	0.0056	0.0074	0.0093	0.0131	0.0192	3.1828

Table 7. Conditional Mean Distance Traveled Given Successful Transit (Simulation).

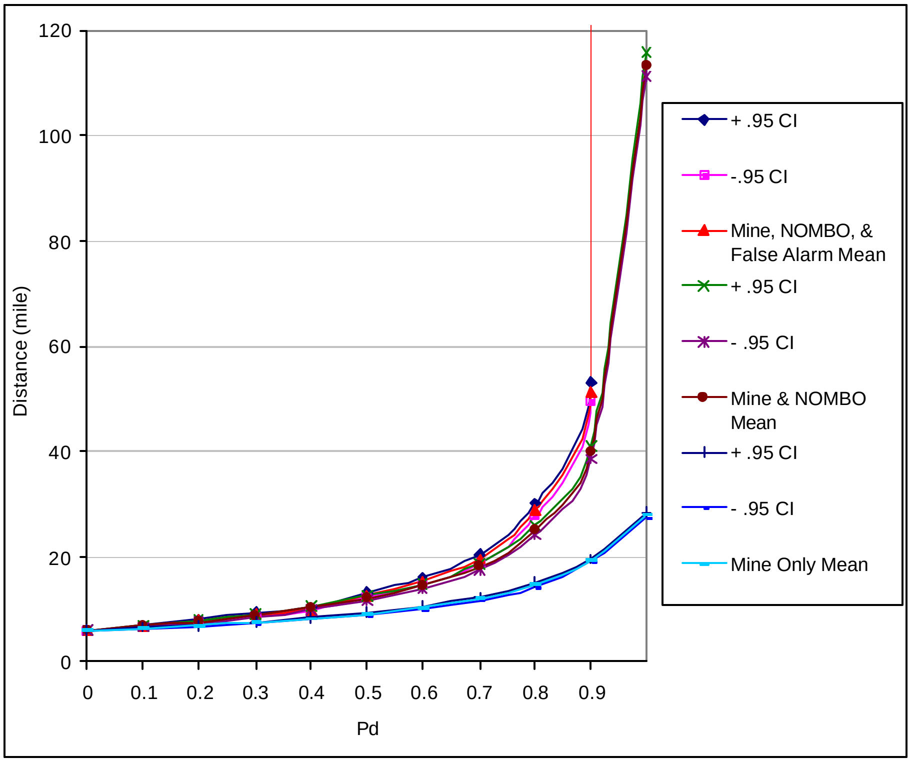

Figure 17. Conditional Mean Distance Traveled Given Successful Transit and 95% CI.

Figures 18 and 19 below display histograms of the distance traveled given successful transit, when there exist mines and NOMBOs in the field and there are no false alarms, and the distance traveled is greater than the distance of the minefield. Two cases (low and high rates of occurrence of NOMBOs in the minefield) are compared to study how the rate of occurrence of NOMBOs influences the minefield travel distance given successful transit. The low (respectively high) rate of occurrence of the NOMBOs is 0.3 (respectively 1.5).

When l_O = 0.3, the fraction of replications that the distance traveled given successful transit equals to the distance of the minefield is 2,095/6,130 = 0.3418; when l_O = 1.5, the fraction of replications in which the distance equals to the distance of the minefield is 170/2,087 = 0.0815. So, for the purpose of analysis, the replications that the distance traveled given successful transit equals to the distance of the minefield are truncated from the original data. The total number of observations displayed in Figure 18 is 4035 and in Figure 19 is 1917. The mean and maximum distances traveled as displayed in Figure 18 are 18.41 miles and 112.30 miles, and in Figure 19 are 38.92 miles and

207.09 miles respectively. This shows us that, when the rate of occurrence of NOMBOs increases in the minefield, the mean and maximum distances traveled also increase. The conditional distribution of the distances traveled is very long tailed. This suggests that even if the ship successfully transits the field, it may take a long time doing so.

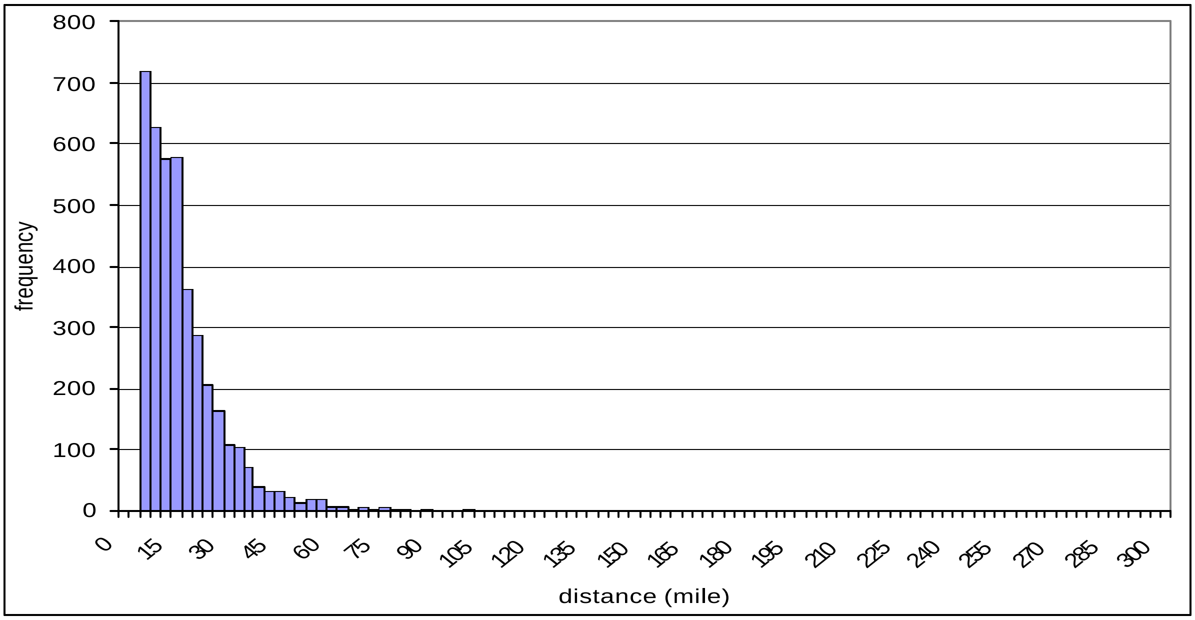

Figure 18. Histogram of Distance Traveled Given Successful Transit, L=6, w=0.5, l_M =0.3, l_O =0.3, l_F =0.0, P_d=0.7.

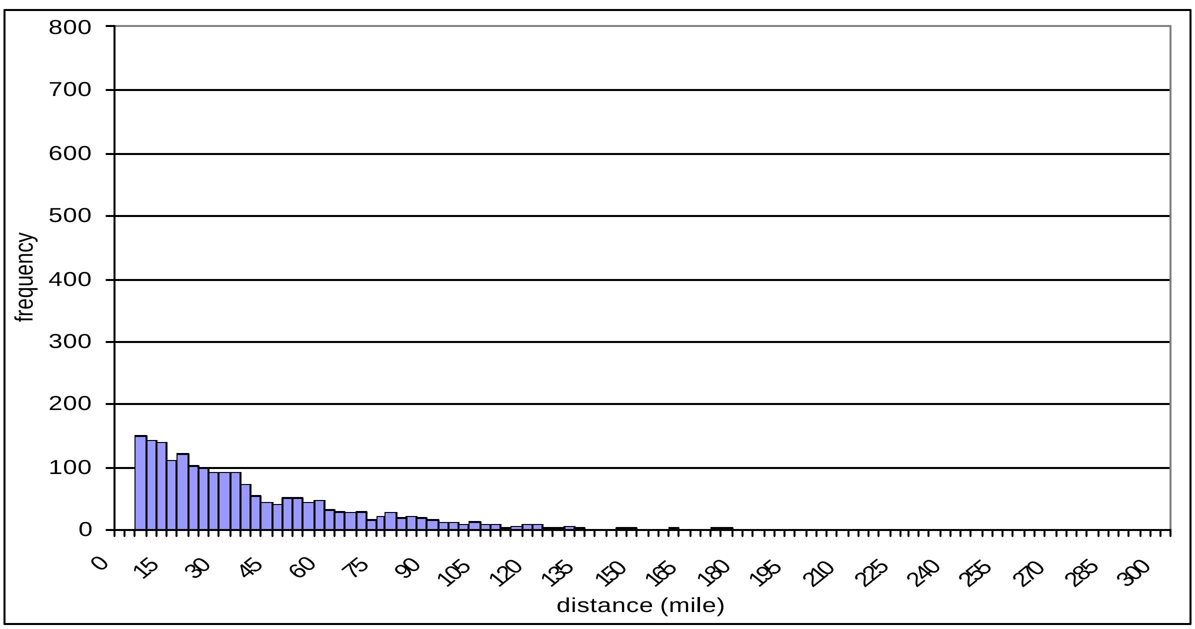

Figure 19. Histogram of Distance Traveled Given Successful Transit, L=6, w=0.5, l_M =0.3, l_O =1.5, l_F =0.0, P_d=0.7.

3. Mean Distance Traveled

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mine only	+.95 CI	2.8363	3.3348	3.8707	4.4893	5.3512	6.4852	7.9455	9.9117	13.053	18.330	28.451
	- .95 CI	2.7561	3.2274	3.7384	4.3319	5.1604	6.2542	7.6568	9.5543	12.582	17.682	27.465
	Mean	2.7962	3.2811	3.8046	4.4106	5.2558	6.3697	7.8012	9.7330	12.817	18.006	27.958
	Std Err	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0007	0.0009	0.0012	0.0017	0.0025
Mine & NOMBO Object	+.95 CI	2.8363	3.5634	4.3807	5.4442	6.8184	8.6212	11.190	15.157	22.159	38.075	115.70
	- .95 CI	2.7561	3.4385	4.2163	5.2313	6.5501	8.2732	10.755	14.576	21.300	36.614	111.25
	Mean	2.7962	3.5009	4.2985	5.3377	6.6843	8.4472	10.972	14.867	21.729	37.345	113.47
	Std Err	0.0002	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011	0.0015	0.0022	0.0037	0.0113
Mine, NOMBO object, & false alarm	+.95 CI	2.8363	3.5979	4.4727	5.5420	6.9697	8.9800	11.695	16.115	25.416	48.946	32488
	- .95 CI	2.7561	3.4723	4.3032	5.3261	6.6943	8.6225	11.231	15.484	24.421	47.065	31240
	Mean	2.7962	3.5351	4.3880	5.4340	6.8320	8.8013	11.463	15.799	24.918	48.005	31864
	Std Err	0.0002	0.0003	0.0004	0.0006	0.0007	0.0009	0.0012	0.0016	0.0025	0.0048	3.18

Table 8. Mean Distance Traveled (Simulation).

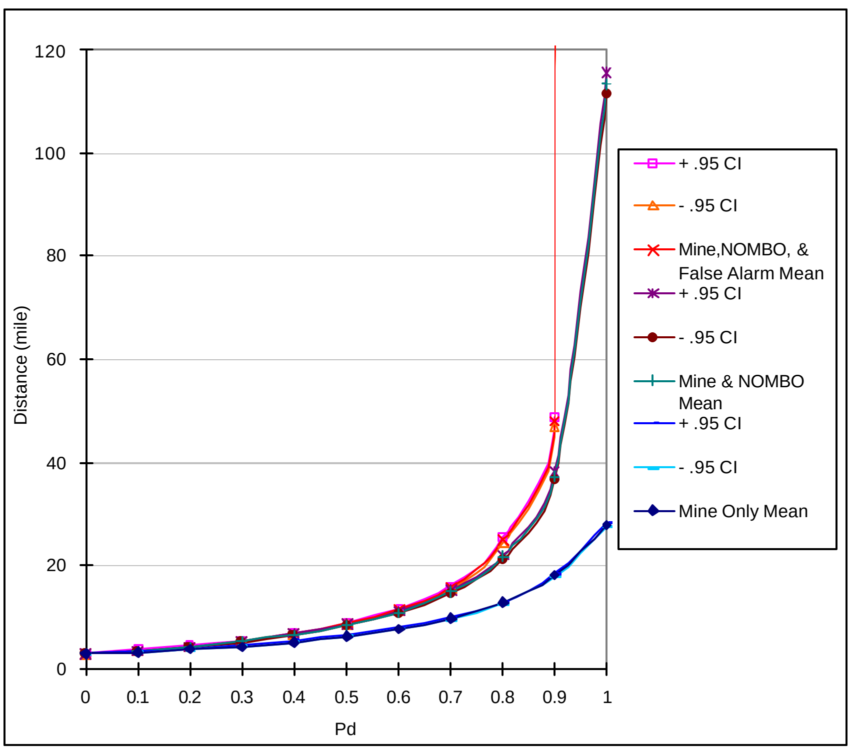

Figure 20. Mean Distance Traveled and 95% CI.

Table 8 and Figure 20 above show the mean distance traveled. This mean distance traveled contains not only the distance traveled given successful transit but also the distance traveled given unsuccessful transit.

4. Mean Number of Retracings

Table 9 and Figure 21 below show the mean number of retracings (returns to the entry to the minefield) according to the probability of detection. The mean number of retracings increases as the probability of detection increases. The shape of Figure 21 is similar to that of Figure 20, indicating a close relation between the distance traveled and the number of retracings (returns to the entry to the minefield).

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mine only	+.95 CI	0.0000	0.1015	0.2149	0.3443	0.5218	0.7495	1.0502	1.4506	2.0940	3.1676	5.2185
	- .95 CI	0.0000	0.0889	0.1955	0.3183	0.4876	0.7055	0.9926	1.3782	1.9950	3.0274	4.9979
	Mean	0.0000	0.0952	0.2052	0.3313	0.5047	0.7275	1.0214	1.4144	2.0445	3.0975	5.1082
	Std Err	0.0000	0.0032	0.0049	0.0066	0.0087	0.0112	0.0147	0.0185	0.0253	0.0358	0.0563
Mine & NOMBO Object	+.95 CI	0.0000	0.1965	0.4365	0.7351	1.1400	1.6838	2.4973	3.7615	6.0080	11.192	36.566
	- .95 CI	0.0000	0.1779	0.4061	0.6911	1.0796	1.6000	2.3839	3.6013	5.7566	10.735	35.098
	Mean	0.0000	0.1872	0.4213	0.7131	1.1098	1.6419	2.4406	3.6814	5.8823	10.964	35.832
	Std Err	0.0000	0.0047	0.0077	0.0112	0.0154	0.0214	0.0289	0.0409	0.0641	0.1165	0.3746
Mine, NOMBO object, & false alarm	+.95 CI	0.0000	0.2056	0.4628	0.7816	1.2366	1.9051	2.8433	4.4486	7.9655	17.772	27640
	- .95 CI	0.0000	0.1868	0.4312	0.7364	1.1724	1.8141	2.7161	4.2598	7.6383	17.068	26578
	Mean	0.0000	0.1962	0.4470	0.7590	1.2045	1.8596	2.7797	4.3542	7.8019	17.420	27109
	Std Err	0.0000	0.0048	0.0081	0.0115	0.0164	0.0232	0.0324	0.0482	0.0834	0.1796	270.81

Table 9. Mean Number of Retracings (Simulation).

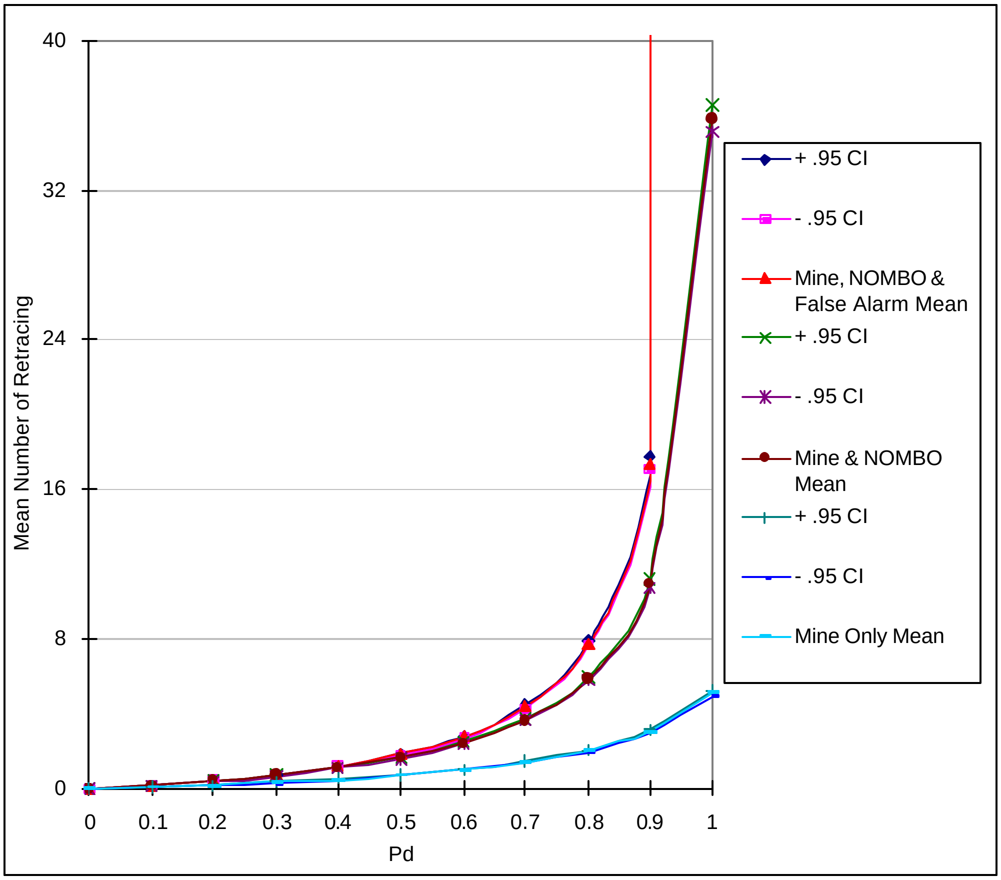

Figure 21. Mean Number of Retracings in the Minefield and 95% CI.

D. DISCUSSION

Using the probability of a safe minefield transit in various environments according to the probability of mine or NOMBO detection as the MOE[8], the simulation output compares well to the analytical SMT model. The analytical calculation results are within the 95% confidence intervals obtained from the simulation outputs with the same input. This suggests the simulation is consistent with the analytical SMT model for these parameters.

[8] MOE: Measure Of Effectiveness.

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IV. EFFECT OF THE NOMBO AND FALSE ALARM ON THE SAFE MINEFIELD TRANSIT IN THE SIMPLE MODEL

A. INTRODUCTION

This chapter explores the effects of the NOMBOs and false alarms on the probability of safe minefield transit in a SMT model. This is accomplished by varying the rate of the occurrence of NOMBOs (l_O) and detection index (*d*)[9] respectively and is compared with a no sensor case when a ship transits the minefield along a direct, straight line without a sensor. If a probability of safe minefield transit (*P(S)*), when using a sensor, is less than or equal to that with no sensor case, there is no benefit to the ship using a sensor to transit the minefield. The rate of occurrence of NOMBOs ranges from 0 to 1.0 in increments of 0.1. The detection index ranges from 0 to 10.0. On the intervals 0 to 1.0, the increments are 0.2. On the intervals 2.0 to 10.0, the increments are 2. The probabilities of a false alarm used in the models appear in Table 48 in Appendix A. The MOE under investigation is the probability of a safe minefield transit, and if and how the change of rate of the occurrence of NOMBOs (l_O) or detection index (*d*) affects this probability. The analytical model in equation (1) is used to obtain the results in this section. However, the results could also have been obtained using the simulation.

B. INPUT PARAMETERS

The table below shows the input parameters used in the Analytical model.

Environment	Minefield Distance (*L*)	Mine Actuation width of ship (*w*)	Rate of Mine (l_M)	Rate of NOMBO (l_0)	Detection Index (*d*)
Mine Only	6	.5	0.1 ~ 1.0	0	-
Mine & NOMBO	6	.5	0.2, 0.6, 1.0	0.0 ~ 1.0	-
Mine & False Alarm	6	.5	0.2, 0.6, 1.0	0	0.0 ~ 10.0
Mine, NOMBO & False Alarm	6	.5	0.2, 0.6, 1.0	.6	0.0 ~ 10.0

Table 10. Input Parameters for Each Environment.

[9] Detection Index affects a rate of false alarm.

C. OUTPUTS

1. Mine Only Case

lambdaM parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.741	0.761	0.781	0.803	0.827	0.851	0.877	0.905	0.935	0.966	1.000
0.2	0.549	0.575	0.603	0.635	0.670	0.709	0.753	0.802	0.859	0.924	1.000
0.3	0.407	0.432	0.461	0.495	0.533	0.578	0.631	0.695	0.774	0.873	1.000
0.4	0.301	0.324	0.350	0.381	0.418	0.463	0.519	0.590	0.683	0.812	1.000
0.5	0.223	0.242	0.264	0.291	0.324	0.365	0.418	0.489	0.590	0.742	1.000
0.6	0.165	0.180	0.198	0.221	0.248	0.284	0.331	0.398	0.498	0.664	1.000
0.7	0.122	0.134	0.149	0.166	0.189	0.218	0.259	0.317	0.411	0.583	1.000
0.8	0.091	0.100	0.111	0.125	0.143	0.166	0.200	0.250	0.333	0.499	1.000
0.9	0.067	0.074	0.083	0.093	0.107	0.126	0.153	0.194	0.265	0.419	1.000
1.0	0.050	0.055	0.061	0.070	0.080	0.095	0.116	0.149	0.208	0.344	1.000

Table 11. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$.

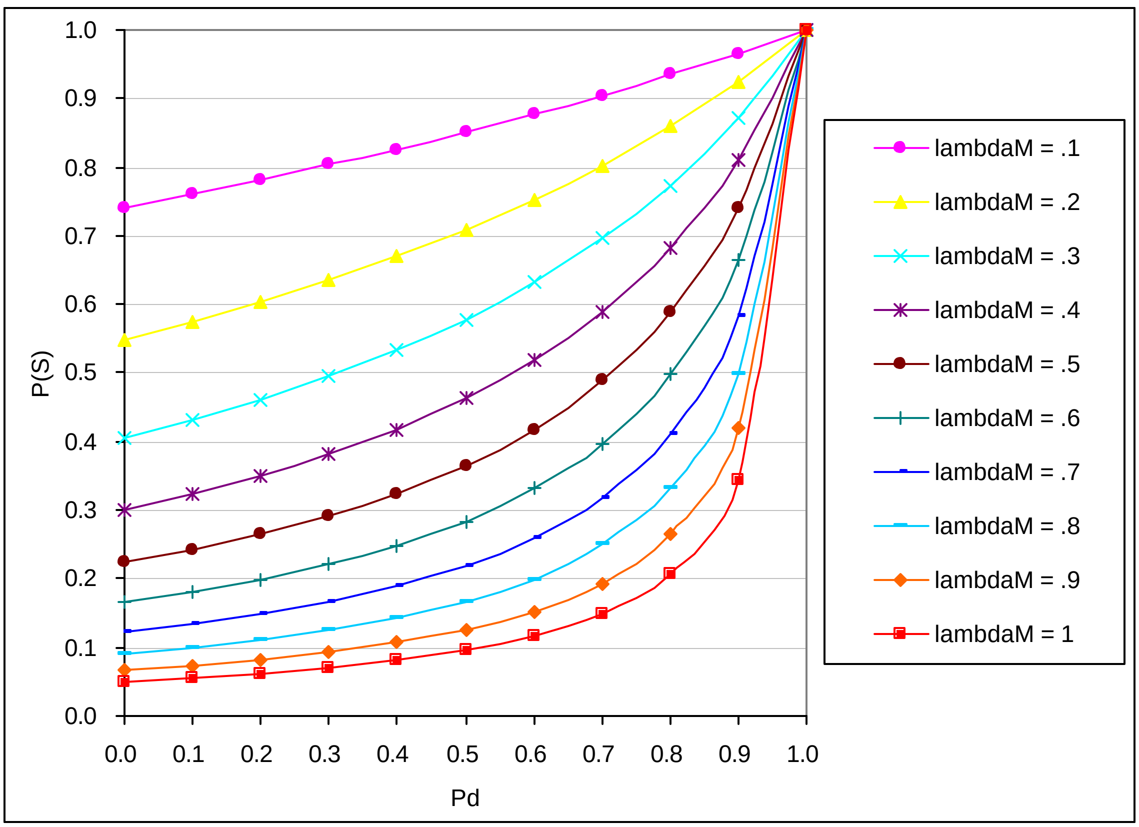

Figure 22. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$.

Table 11 and Figure 22 above are generated from the results of the analytical models. The probabilities of a safe minefield transit are plotted. Table 11 shows the probabilities of successful minefield transit for various l_M. When the probability of detection increases and the rate of occurrence of deployed mines decreases, the probability of safe minefield transit increases. In addition, there is no probability of a safe minefield transit with a sensor that is less than that of a no sensor case, which means that, whenever a ship uses a sensor, the probability of safe minefield transit never decreases. Thus, it is beneficial to the ship to use a sensor, even when the probability of detection of the sensor is low, while transiting the minefield in this case in which there are no NOMBOs and no false alarms.

2. Mine and NOMBO Case

The tables below show the probability of safe minefield transit, when the rate of occurrence of mines is 0.2, 0.6, and 1.0, respectively. The rate of occurrence of NOMBOs ranges from 0.0 to 1.0 in increments of 0.1. There are no false alarms. The effect of the rate of occurrence of NOMBOs on the probability of a safe minefield transit is shown as the rate of the occurrence of mines in the field increases. The case with probability of detection equal to 0 is that a ship does not use a sensor and transits the minefield in a straight line.

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.549	0.575	0.603	0.635	0.670	0.709	0.753	0.802	0.859	0.924	1.000
0.1	0.549	0.571	0.595	0.623	0.655	0.691	0.733	0.783	0.842	0.913	1.000
0.2	0.549	0.567	0.587	0.611	0.639	0.673	0.713	0.762	0.822	0.899	1.000
0.3	0.549	0.563	0.579	0.599	0.623	0.653	0.691	0.738	0.800	0.884	1.000
0.4	0.549	0.558	0.571	0.586	0.607	0.633	0.667	0.713	0.776	0.865	1.000
0.5	0.549	0.554	0.562	0.574	0.590	0.612	0.642	0.685	0.748	0.844	1.000
0.6	0.549	0.550	0.554	0.561	0.572	0.590	0.616	0.656	0.718	0.818	1.000
0.7	0.549	**0.546**	**0.545**	**0.547**	0.554	0.567	0.588	0.624	0.684	0.790	1.000
0.8	0.549	**0.542**	**0.536**	**0.534**	**0.535**	**0.543**	0.560	0.591	0.648	0.757	1.000
0.9	0.549	**0.537**	**0.528**	**0.520**	**0.517**	**0.519**	**0.530**	0.556	0.608	0.719	1.000
1.0	0.549	**0.533**	**0.519**	**0.506**	**0.498**	**0.494**	**0.499**	**0.519**	0.567	0.678	1.000

Table 12. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_M=0.2$.

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	0.180	0.198	0.221	0.248	0.284	0.331	0.398	0.498	0.664	1.000
0.1	0.165	0.178	0.192	0.211	0.234	0.264	0.306	0.365	0.459	0.624	1.000
0.2	0.165	0.175	0.186	0.201	0.220	0.246	0.281	0.334	0.419	0.581	1.000
0.3	0.165	0.172	0.180	0.192	0.207	0.228	0.257	0.303	0.380	0.535	1.000
0.4	0.165	0.169	0.175	0.183	0.194	0.210	0.234	0.273	0.341	0.488	1.000
0.5	0.165	0.166	0.169	0.174	0.181	0.194	0.213	0.245	0.304	0.439	1.000
0.6	0.165	**0.164**	**0.164**	**0.165**	0.170	0.178	0.192	0.218	0.268	0.391	1.000
0.7	0.165	**0.161**	**0.158**	**0.157**	**0.158**	**0.163**	0.173	0.193	0.235	0.344	1.000
0.8	0.165	**0.158**	**0.153**	**0.149**	**0.147**	**0.149**	**0.155**	0.170	0.204	0.299	1.000
0.9	0.165	**0.156**	**0.148**	**0.141**	**0.137**	**0.135**	**0.138**	**0.149**	0.176	0.258	1.000
1.0	0.165	**0.153**	**0.143**	**0.134**	**0.127**	**0.123**	**0.123**	**0.130**	**0.151**	0.219	1.000

Table 13. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_M=0.6$.

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.055	0.061	0.070	0.080	0.095	0.116	0.149	0.208	0.344	1.000
0.1	0.050	0.054	0.059	0.066	0.074	0.086	0.103	0.130	0.180	0.301	1.000
0.2	0.050	0.053	0.057	0.062	0.068	0.078	0.092	0.114	0.156	0.261	1.000
0.3	0.050	0.052	0.054	0.058	0.063	0.070	0.081	0.099	0.133	0.223	1.000
0.4	0.050	0.051	0.052	0.054	0.058	0.063	0.071	0.086	0.114	0.190	1.000
0.5	0.050	**0.050**	**0.050**	0.051	0.053	0.057	0.063	0.074	0.096	0.159	1.000
0.6	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.081	0.133	1.000
0.7	0.050	**0.048**	**0.046**	**0.045**	**0.045**	**0.046**	**0.048**	0.054	0.068	0.110	1.000
0.8	0.050	**0.047**	**0.044**	**0.042**	**0.041**	**0.041**	**0.042**	**0.046**	0.057	0.090	1.000
0.9	0.050	**0.046**	**0.042**	**0.039**	**0.038**	**0.037**	**0.037**	**0.040**	**0.047**	0.074	1.000
1.0	0.050	**0.045**	**0.040**	**0.037**	**0.034**	**0.033**	**0.032**	**0.034**	**0.039**	0.060	1.000

Table 14. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_M=1.0$.

The colored boxes in the above tables indicate that the probability of a safe minefield transit is less than or equal to that of a no sensor case.

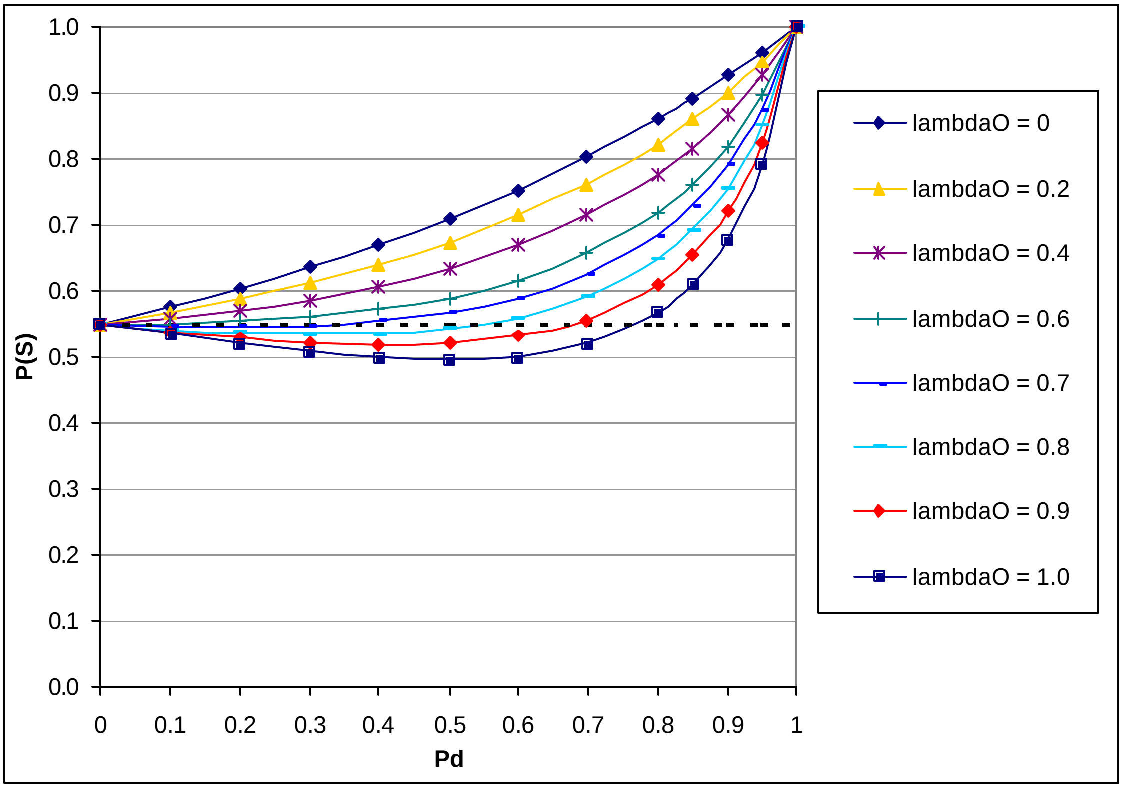

Figure 23. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_M=0.2$.

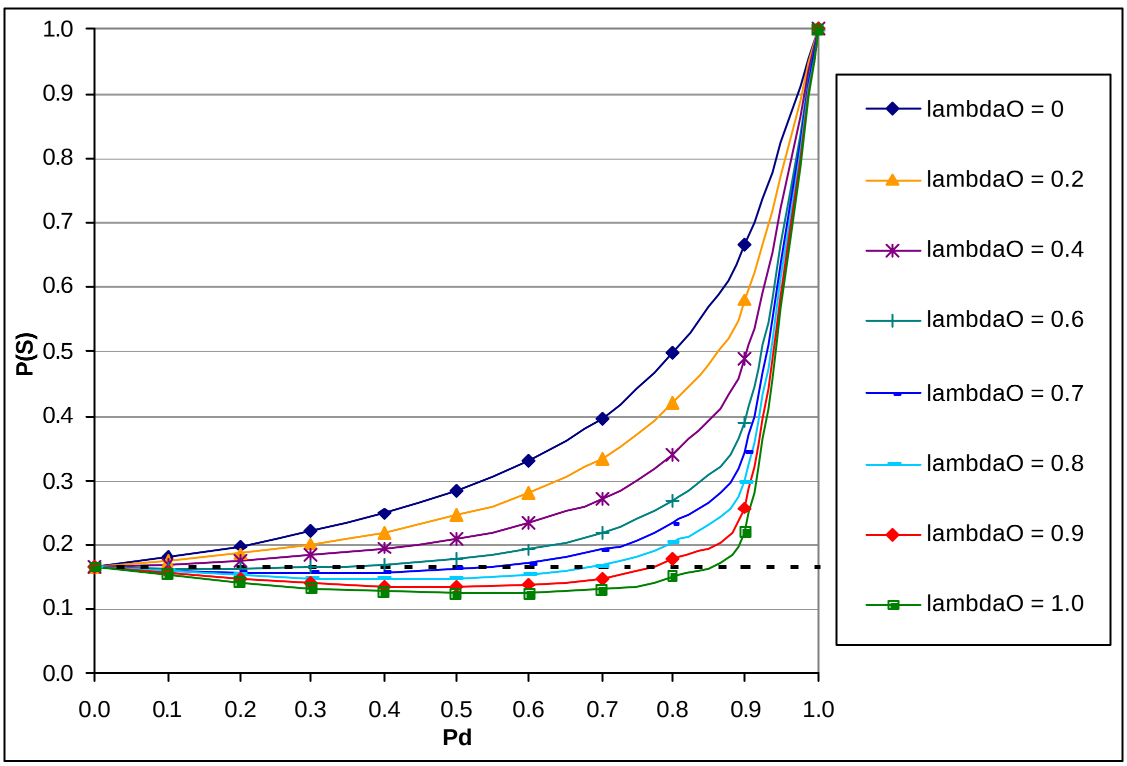

Figure 24. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_M=0.6$.

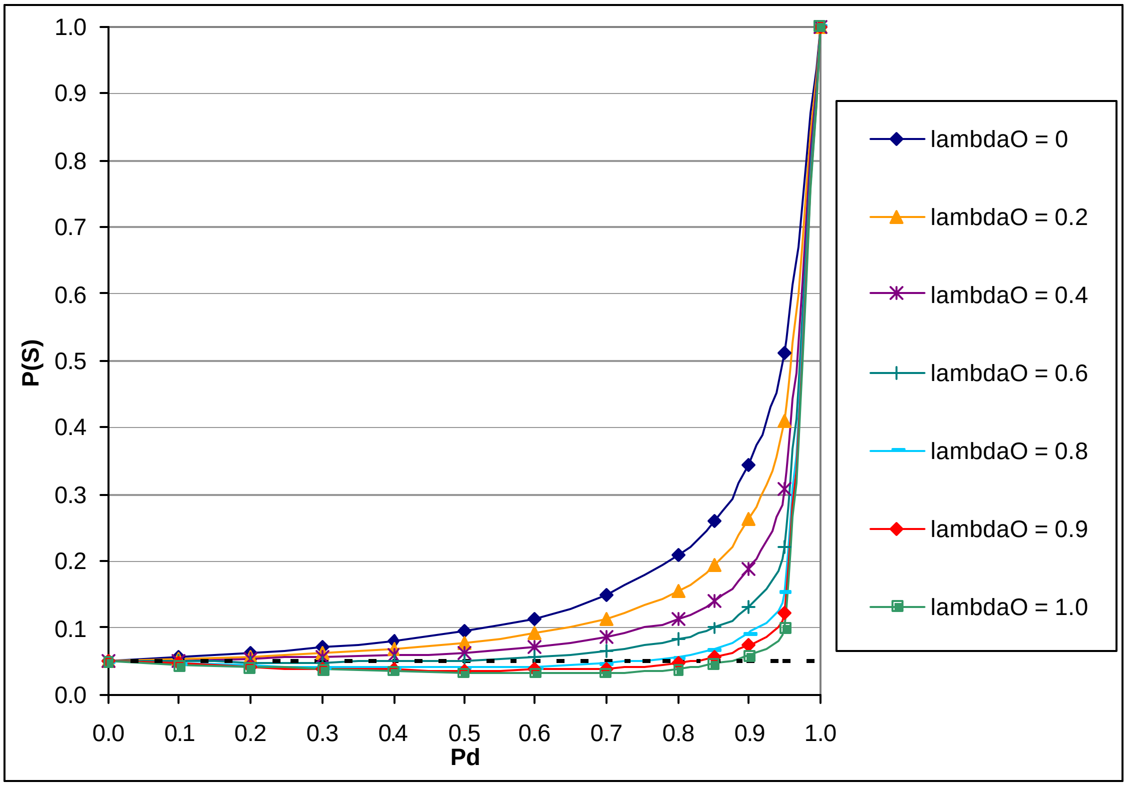

Figure 25. Probability of Safe Minefield Transit, *L=6*, *w=0.5*, l_M *=1.0*.

All the values located below the dotted line in the above graphs are the probabilities of a safe minefield transit which are less than or equal to that of the no sensor case. The tables and graphs clearly indicate a pattern. The plots of the data above show that a relation exists between the probability of a safe minefield transit and the rate of occurrence of NOMBOs in the minefield. In addition, it is possible to determine that, as the rate of occurrence of mines increases, the region in which the probability of a safe minefield transit when using a sensor falls below that of the no sensor case also increases.

3. Mine and False Alarm Case

The tables below show the probabilities of a safe minefield transit, when the rate of occurrence of NOMBOs is 0.0 and the rate of occurrence of mines is 0.2, 0.6, and 1.0, respectively. In addition, the rate of occurrence of false alarms is a function of the probability of detection and the detection index as discussed in the appendix A. The detection index ranges from 0.0 to 10.0. This shows the effect of a false alarm on the

probability of a safe minefield transit according to the rate of occurrence of mines in the field.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.549	0.561	0.573	0.589	0.607	0.620	0.640	0.668	0.702	0.745	1.000
0.2	0.549	0.569	0.590	0.612	0.637	0.661	0.690	0.726	0.770	0.825	1.000
0.4	0.549	0.571	0.594	0.619	0.644	0.673	0.705	0.744	0.786	0.848	1.000
0.6	0.549	0.572	0.597	0.622	0.650	0.680	0.715	0.755	0.799	0.858	1.000
0.8	0.549	0.573	0.598	0.625	0.653	0.685	0.720	0.761	0.810	0.870	1.000
1.0	0.549	0.573	0.599	0.627	0.657	0.688	0.725	0.767	0.817	0.877	1.000
2.0	0.549	0.574	0.602	0.631	0.664	0.700	0.740	0.784	0.835	0.897	1.000
4.0	0.549	0.575	0.603	0.634	0.668	0.706	0.748	0.796	0.850	0.913	1.000
6.0	0.549	0.575	0.603	0.635	0.669	0.708	0.751	0.800	0.855	0.919	1.000
8.0	0.549	0.575	0.603	0.635	0.670	0.708	0.752	0.801	0.857	0.922	1.000
10.0	0.549	0.575	0.603	0.635	0.670	0.709	0.752	0.802	0.858	0.923	1.000

Table 15. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=0.2$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	0.171	0.177	0.184	0.194	0.200	0.211	0.228	0.253	0.286	1.000
0.2	0.165	0.176	0.188	0.202	0.218	0.234	0.257	0.288	0.333	0.403	1.000
0.4	0.165	0.178	0.191	0.207	0.224	0.246	0.273	0.310	0.356	0.449	1.000
0.6	0.165	0.179	0.193	0.209	0.229	0.253	0.283	0.325	0.378	0.470	1.000
0.8	0.165	0.179	0.194	0.212	0.232	0.258	0.290	0.333	0.397	0.500	1.000
1.0	0.165	0.179	0.195	0.214	0.236	0.261	0.295	0.341	0.408	0.517	1.000
2.0	0.165	0.180	0.197	0.218	0.243	0.274	0.314	0.366	0.445	0.574	1.000
4.0	0.165	0.180	0.198	0.220	0.247	0.281	0.325	0.387	0.478	0.626	1.000
6.0	0.165	0.180	0.198	0.220	0.248	0.283	0.329	0.394	0.489	0.647	1.000
8.0	0.165	0.180	0.198	0.221	0.248	0.283	0.330	0.396	0.494	0.655	1.000
10.0	0.165	0.180	0.198	0.221	0.248	0.284	0.331	0.397	0.496	0.660	1.000

Table 16. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=0.6$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.051	0.053	0.055	0.058	0.059	0.062	0.068	0.075	0.085	1.000
0.2	0.050	0.054	0.057	0.062	0.068	0.073	0.081	0.092	0.110	0.139	1.000
0.4	0.050	0.054	0.059	0.064	0.070	0.078	0.088	0.102	0.121	0.165	1.000
0.6	0.050	0.054	0.059	0.065	0.072	0.081	0.093	0.109	0.132	0.178	1.000
0.8	0.050	0.054	0.060	0.066	0.073	0.083	0.095	0.114	0.143	0.198	1.000
1.0	0.050	0.055	0.060	0.067	0.075	0.085	0.098	0.118	0.149	0.210	1.000
2.0	0.050	0.055	0.061	0.068	0.078	0.090	0.107	0.131	0.172	0.255	1.000
4.0	0.050	0.055	0.061	0.069	0.080	0.093	0.113	0.142	0.193	0.303	1.000
6.0	0.050	0.055	0.061	0.070	0.080	0.094	0.115	0.146	0.202	0.324	1.000
8.0	0.050	0.055	0.061	0.070	0.080	0.095	0.115	0.148	0.205	0.334	1.000
10.0	0.050	0.055	0.061	0.070	0.080	0.095	0.116	0.148	0.206	0.339	1.000

Table 17. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=1.0$.

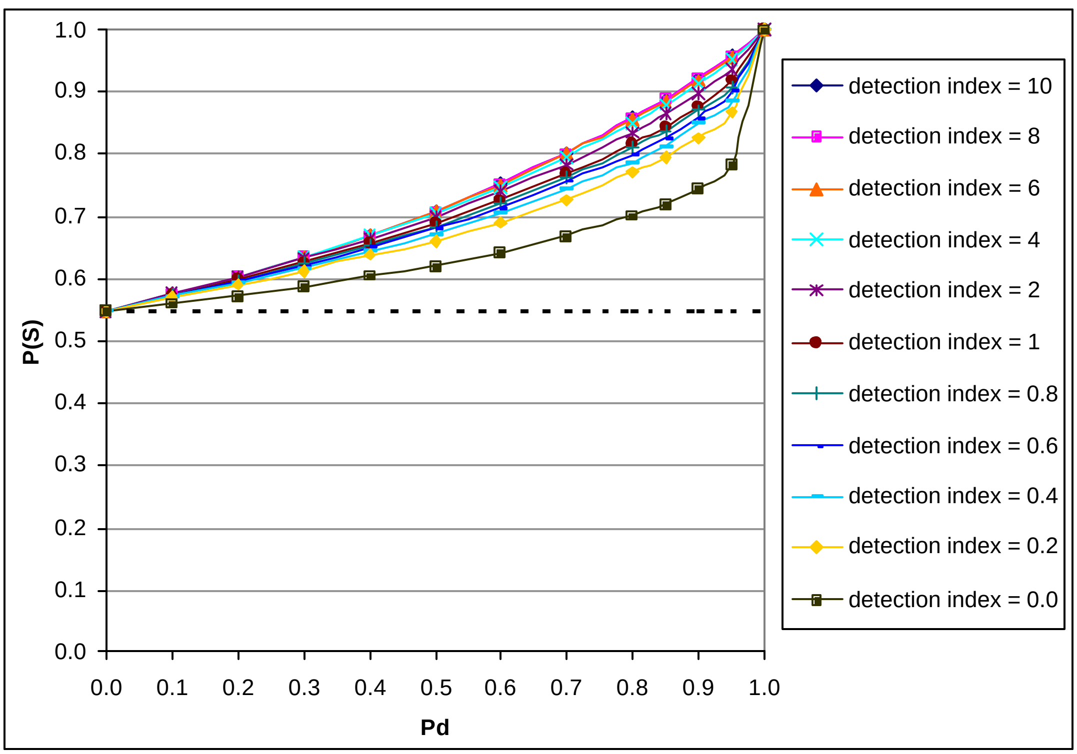

Figure 26. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=0.2$.

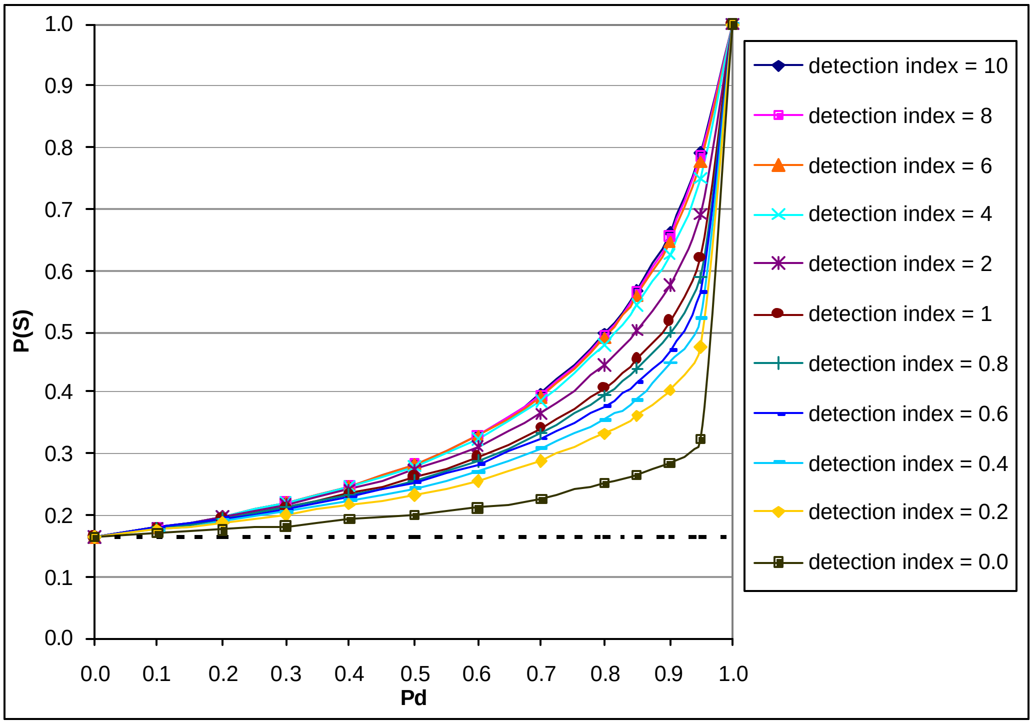

Figure 27. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=0.6$.

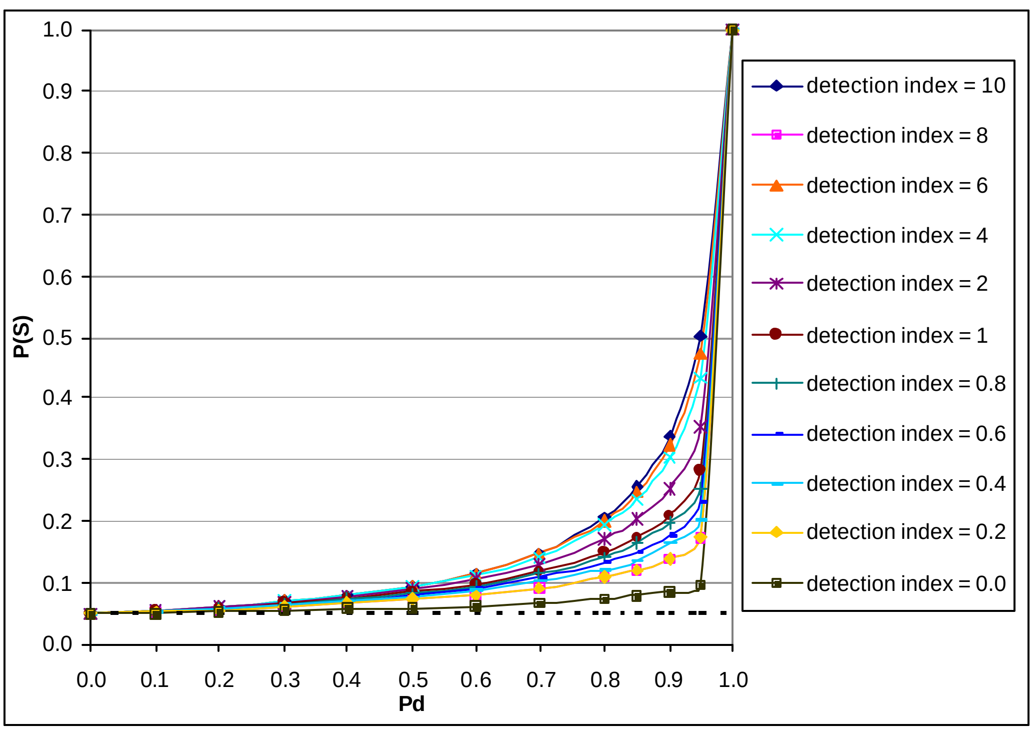

Figure 28. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$, $l_M=1.0$.

The tables and graphs clearly show the effect of false alarms on the probability of a safe minefield transit. As the detection index increases and the rate of occurrence of false alarms decreases, the probability of safe minefield transit increases. For a fixed probability of detection, the probability of a safe minefield transit is essentially constant for detection indices greater than 4. The probability of a safe minefield transit with a sensor is always greater than the probability of a safe minefield transit in a no sensor case.

4. Mine, NOMBO, and False Alarm Case

The tables below show the probabilities of a safe minefield transit when the rate of the occurrence of NOMBOs is 0.6 and the rate of occurrence of mines is 0.2, 0.6, and 1.0 respectively, and the rate of occurrence of false alarms is a function of the probability of detection and the detection index as described in Appendix A. These results shows the effect of NOMBOs and false alarms on the probability of a safe minefield transit as a function of the rate of occurrence of mines in the field.

Detection Index (*d*)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.549	**0.536**	**0.522**	**0.509**	**0.497**	**0.479**	**0.466**	**0.460**	**0.459**	**0.464**	1.000
0.2	0.549	**0.544**	**0.539**	**0.535**	**0.533**	**0.528**	**0.529**	**0.538**	0.558	0.597	1.000
0.4	0.549	**0.546**	**0.544**	**0.543**	**0.541**	**0.543**	0.549	0.564	0.584	0.643	1.000
0.6	0.549	**0.548**	**0.546**	**0.546**	**0.548**	0.553	0.562	0.581	0.606	0.662	1.000
0.8	0.549	**0.548**	**0.548**	0.549	0.552	0.558	0.570	0.590	0.626	0.689	1.000
1.0	0.549	**0.549**	0.549	0.551	0.557	0.563	0.576	0.599	0.637	0.704	1.000
2.0	0.549	0.550	0.552	0.557	0.565	0.578	0.597	0.625	0.672	0.751	1.000
4.0	0.549	0.550	0.553	0.560	0.570	0.586	0.610	0.646	0.701	0.791	1.000
6.0	0.549	0.550	0.554	0.560	0.572	0.588	0.614	0.652	0.711	0.806	1.000
8.0	0.549	0.550	0.554	0.561	0.572	0.589	0.615	0.654	0.714	0.812	1.000
10.0	0.549	0.550	0.554	0.561	0.572	0.590	0.616	0.655	0.716	0.816	1.000

Table 18. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=0.2$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	**0.155**	**0.144**	**0.135**	**0.127**	**0.116**	**0.108**	**0.103**	**0.100**	**0.098**	1.000
0.2	0.165	**0.160**	**0.154**	**0.150**	**0.146**	**0.140**	**0.138**	**0.139**	**0.146**	**0.162**	1.000
0.4	0.165	**0.161**	**0.157**	**0.154**	**0.151**	**0.149**	**0.149**	**0.154**	**0.160**	0.192	1.000
0.6	0.165	**0.162**	**0.159**	**0.156**	**0.155**	**0.154**	**0.157**	**0.164**	0.175	0.207	1.000
0.8	0.165	**0.162**	**0.160**	**0.158**	**0.157**	**0.158**	**0.161**	0.170	0.188	0.228	1.000
1.0	0.165	**0.163**	**0.161**	**0.159**	**0.160**	**0.161**	**0.165**	0.175	0.196	0.243	1.000
2.0	0.165	**0.163**	**0.163**	**0.163**	**0.165**	0.170	0.179	0.194	0.224	0.293	1.000
4.0	0.165	**0.164**	**0.163**	**0.165**	0.169	0.175	0.188	0.210	0.251	0.346	1.000
6.0	0.165	**0.164**	**0.164**	**0.165**	0.169	0.177	0.191	0.215	0.261	0.370	1.000
8.0	0.165	**0.164**	**0.164**	**0.165**	0.170	0.178	0.192	0.217	0.265	0.380	1.000
10.0	0.165	**0.164**	**0.164**	**0.165**	0.170	0.178	0.192	0.218	0.267	0.386	1.000

Table 19. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=0.6$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	**0.045**	**0.041**	**0.037**	**0.034**	**0.030**	**0.028**	**0.026**	**0.024**	**0.023**	1.000
0.2	0.050	**0.047**	**0.045**	**0.042**	**0.040**	**0.038**	**0.037**	**0.036**	**0.038**	**0.041**	1.000
0.4	0.050	**0.048**	**0.046**	**0.044**	**0.042**	**0.041**	**0.040**	**0.041**	**0.042**	0.051	1.000
0.6	0.050	**0.048**	**0.046**	**0.045**	**0.044**	**0.043**	**0.043**	**0.044**	**0.047**	0.056	1.000
0.8	0.050	**0.048**	**0.047**	**0.045**	**0.044**	**0.044**	**0.044**	**0.046**	0.051	0.063	1.000
1.0	0.050	**0.048**	**0.047**	**0.046**	**0.045**	**0.045**	**0.046**	**0.048**	0.054	0.068	1.000
2.0	0.050	**0.049**	**0.048**	**0.047**	**0.047**	**0.048**	0.051	0.055	0.064	0.088	1.000
4.0	0.050	**0.049**	**0.048**	**0.048**	**0.048**	0.050	0.054	0.060	0.074	0.111	1.000
6.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.062	0.078	0.122	1.000
8.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.080	0.127	1.000
10.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.081	0.130	1.000

Table 20. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

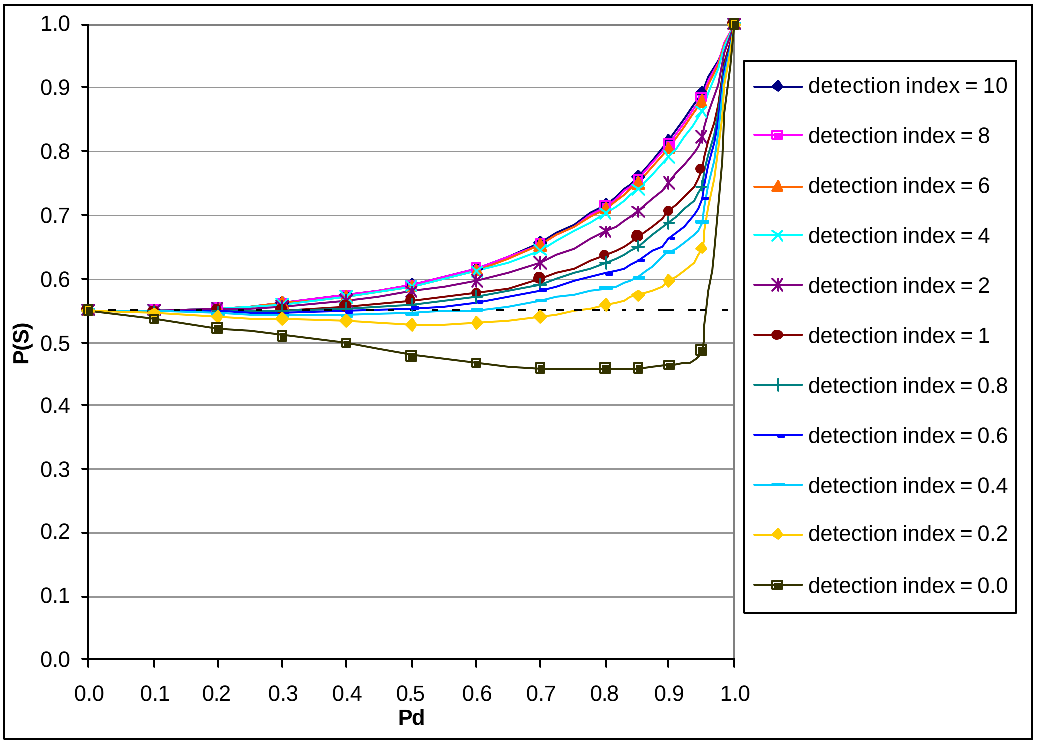

Figure 29. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=0.2$.

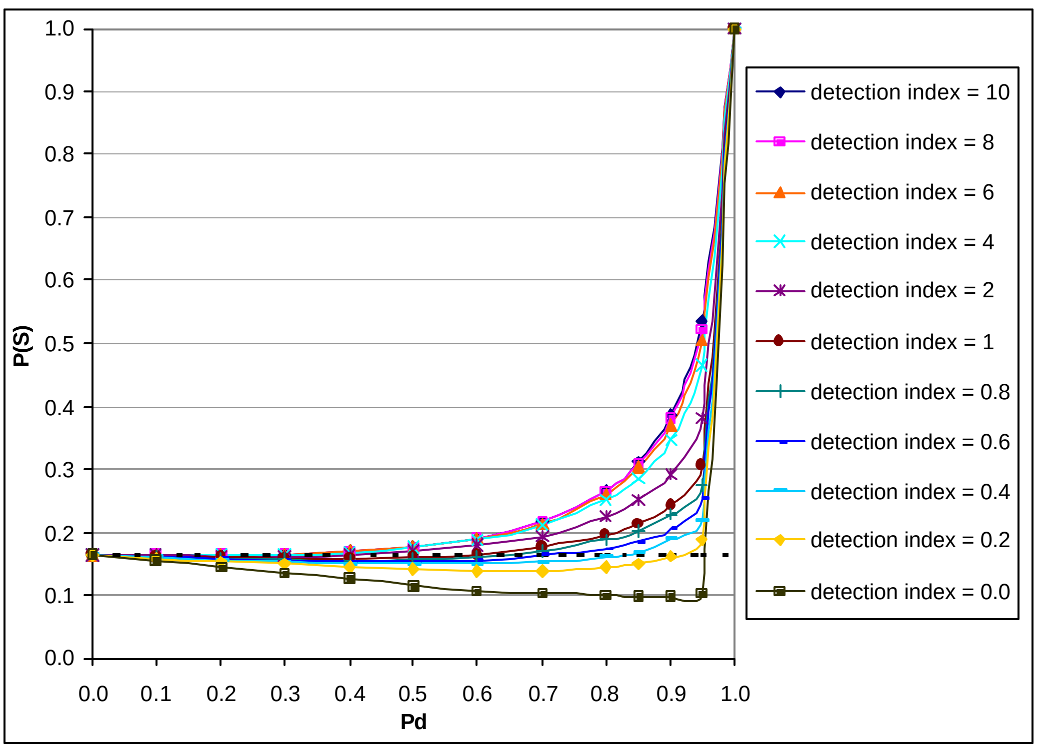

Figure 30. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=0.6$.

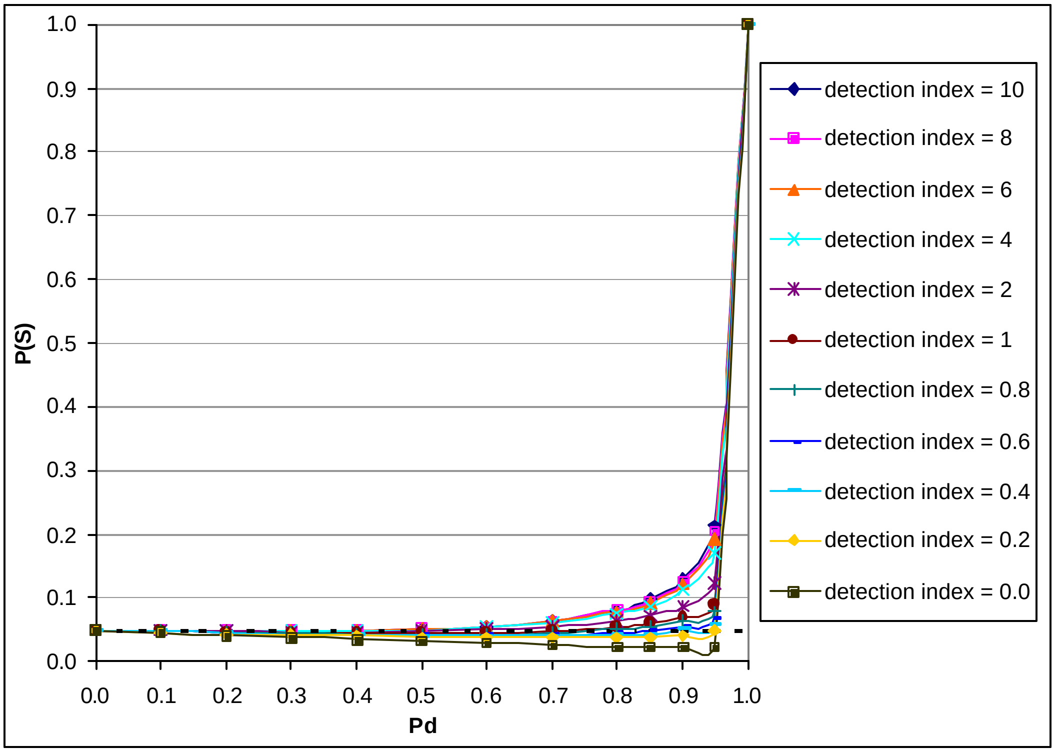

Figure 31. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

The colored boxes in the above tables and all the values located below the dotted line in the above graphs indicate that the probability of a safe minefield transit with a sensor is less than or equal to that with no sensor. The tables above show that, as the detection index increases, the range of sensor detection probabilities for which using a sensor results in a smaller probability of a safe minefield transit through the field than that for a no sensor case becomes smaller.

D. DISCUSSION

Comparing the four cases, it appears that the probability of a safe minefield transit is highly dependent on the rate of occurrence of mines $\left(l_M\right)$ in the minefield. The case that the probability of a safe minefield transit with using a sensor is below that with using no sensor happens when NOMBOs exist in the field. The results also show that, for a fixed probability of detection, the probability of a safe minefield transit is essentially constant for detection indices greater than 4.

In this study, the step of the classification of an object that is detected is not considered. Thus, when the ship detects something in the minefield, it just returns to the entry to the field without classification. However, the results of the mine only case and the mine + false alarm case can be used to study the advantage of having perfect classification capability for mines and NOMBOs. When the ship has a sensor that classifies objects perfectly, even though NOMBOs exist in the minefield, the ship can classify those correctly and continue to proceed as if they were not there. Thus, the results for perfect classification are the same as those of the mine only case and the mine + false alarm case. For instance, in Figure 12, when the probability of detection is 0.7, the probability of safe minefield transit of mine only case is 0.398, where as that of mine and NOMBO case is 0.218. Here, the mine only case can be considered as a perfect classification case, and the mine and NOMBO case can be considered as a no classification case. As you see from the results, when the ship has a perfect classification sensor, the probability of safe minefield is higher than that with no classification sensor; perfect classification results in an approximate 78% increase in the probability of safe transit over having no capability for classification. It is also noted that increasing the probability of detection from 0.7 to 0.9 in this mine and NOMBO case results in a probability of safe passage of 0.391, which is roughly equivalent to the improvement possible by adding perfect classification capability. In this manner, the model can be used to quantify the benefits of alternative investments in either technology that would increase the probability of detection or technology that adds significant classification capability. These quantified benefits could then be used in a complete cost-benefit analysis.

V. MINEFIELD OBJECT AVOIDANCE MANEUVER MODEL

A. ANALYTICAL MODEL

Minefield Object Avoidance Maneuver (MOAM) stochastic models are presented in this section (Jacobs, 2002). The avoidance tactic of the MOAM model is somewhat more complicated than that of the SMT model. The ship is attempting to cross a minefield of distance L across.

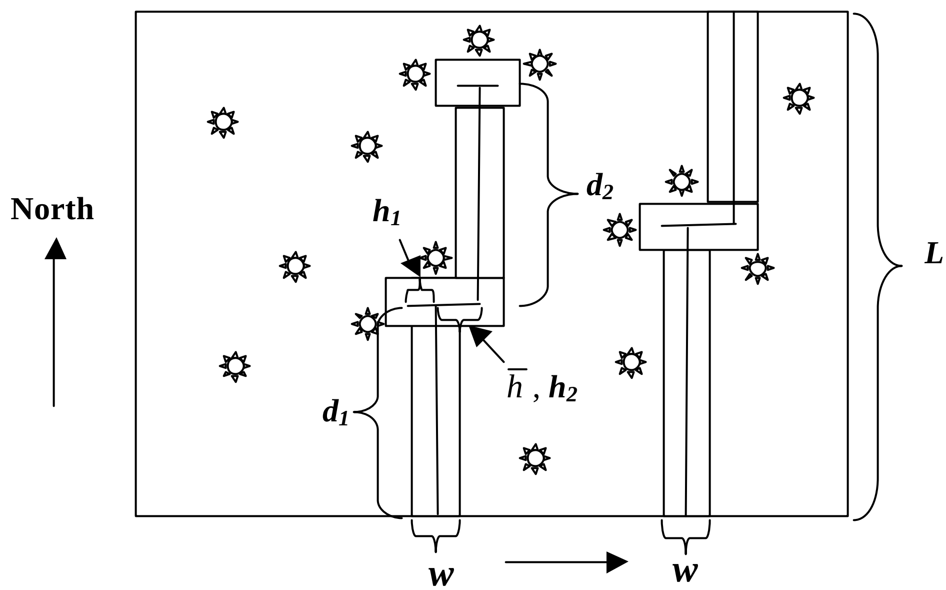

Figure 32. Minefield Transit in MOAM Model.

When the ship encounters a detected mine or NOMBO, or a sensor gives a false alarm, the ship attempts to go around these. The ship goes a distance $\bar{h}$ to the right (for illustration, could alternatively go to the left). If the ship does not detect a mine or NOMBO, or the sensor does not give a false alarm and survives the distance, then it once again proceeds to the end of the field. If the ship encounters a detected object or mine while going to the right, it backtracks and tries a distance $\bar{h}$ to the left. If it does not detect an object and survives during this avoidance path, it once again proceeds to the end of the field. If the ship encounters a detected mine or NOMBO in both directions and survives, it goes back to the entry to the field and starts over again.

l_M, l_O, l_F are the rate of occurrence of mines, rate of occurrence of NOMBOs and rate of occurrence of false alarms per unit area respectively.

The probability that the ship finds nothing in (0, $\bar{h}$) is

$$e^{-\left\{(l_M P_d + l_O P_d + l_F)*\bar{h}*w\right\}} e^{-l_M(1-P_d)*\bar{h}*w}$$
$$= e^{-\left\{(l_M P_d + l_O P_d + l_F)*\bar{h}*w\right\}}$$

The probability that the ship finds something in (0, $\bar{h}$) and survives is

$$\int_0^{\bar{h}} \left\{(l_M P_d + l_O P_d + l_F)w\right\} e^{-\left\{(l_M P_d + l_O P_d + l_F)w\right\}x} e^{-l_M w(1-P_d)x} dx$$
$$= \frac{\left\{(l_M + l_O)P_d + l_F\right\}}{\left\{l_M + l_O P_d + l_F\right\}} \left[1 - e^{-(l_M + l_O P_d + l_F)*\bar{h}*w}\right]$$

Let $g(\bar{h})$ be the probability that the ship of effective width w survives a diversion around a detected mine or NOMBO, but must go back to the entry to the minefield.

$$g\left(\bar{h}\right) = \left[\int_0^{\bar{h}} \left\{(l_M P_d + l_O P_d + l_F)w\right\} e^{-\left\{(l_M P_d + l_O P_d + l_F)w\right\}x} e^{-l_M w(1-P_d)x} dx\right]^2$$
$$= \left[\frac{\left\{(l_M + l_O)P_d + l_F\right\}}{\left\{l_M + l_O P_d + l_F\right\}} \left\{1 - e^{-(l_M + l_O P_d + l_F)*\bar{h}*w}\right\}\right]^2$$

Let $b(\bar{h})$ be the probability that the ship of effective width w survives a diversion around a detected mine or NOMBO and can continue towards the end of the minefield without returning to the entry to the minefield.

$$b\left(\bar{h}\right) = \underbrace{e^{-(l_M + l_O P_d + l_F)*\bar{h}*w}}_{\text{find nothing in first avoiding direction}}$$
$$+ \underbrace{\frac{\left\{(l_M + l_O)P_d + l_F\right\}}{\left\{l_M + l_O P_d + l_F\right\}} \left\{1 - e^{-(l_M + l_O P_d + l_F)*\bar{h}*w}\right\}}_{\text{find something in first avoiding direction}} \underbrace{e^{-(l_M + l_O P_d + l_F)*\bar{h}*w}}_{\text{find nothing in second avoiding direction}}$$

Let f be the probability that an encountered object is not an undetected mine.

$$f = \frac{l_M P_d + l_O P_d + l_F}{l_M + l_O P_d + l_F}$$

Let $P(S)$ be the probability of a successful transit of the minefield.

$$P(S) = \frac{e^{-[l_O P_d w + l_F w][1-b(\bar{h})]L} e^{-l_M wL[1-P_d b(\bar{h})]}}{1-\left[\frac{f * g(\bar{h})}{1-f*b(\bar{h})}\right]\left[1-e^{-(l_O P_d w + l_F w)(1-b(\bar{h}))L} e^{-l_M wL(1-P_d b(\bar{h}))}\right]} \qquad (3)$$

As a special case, when $P_d = 0$

$$P(S) = e^{-l_M wL}$$

This is the probability that the ship survives transit when it does not use a sensor.

B. SIMULATION

The basic function of a MOAM model simulation is similar to the SMT model simulation. A MOAM model simulation has the same input parameters and provides the same kinds of outputs. The simulation results for probability of success are compared with the analytical model results for verification of both formulations. In addition, the simulation enhances the analytical results by providing additional information shown such as conditional mean distance traveled given unsuccessful transit; the conditional mean distance traveled given transit is successful; the mean distance traveled; and the mean number of retracings (returns to the entry to the minefield). The simulation also provides the distribution of the distance traveled in the minefield and counts of path retracing.

Figures 33, 34, and 35 below show pseudo code of the MOAM model simulation. The simulation starts from the Proceeding Tactic. First, draw the distance to the first mine, NOMBO, and false alarm. If the shortest distance among these is greater than L, the loop finishes. Or, according to the probability of detecting mine and NOMBO, and the occurrence of a false alarm, the ship will execute the first avoiding tactic (travel to the right to avoid the object) or be exploded. The second avoiding tactic (travel in the other direction to avoid the object) is executed when the first avoiding tactic is unsuccessful

(encounter an object and detect it) and the ship is not exploded by the mine. If the second avoiding tactic is also unsuccessful and the ship is not exploded by the mine, the ship will go back to the entry to the field and start over again. To obtain output statistics with small standard errors, the simulation is replicated 10,000 times in each run.

Distance need is L (Y axis)
Width of mine actuation is w

l_M = E[# mines / unit area]
l_O = E[# of NOMBOs / unit area]
l_F = E[# false Alarms / unit area]

$P_d(M)$ = Probability of detecting mine
$P_d(O)$ = Probability of detecting NOMBO

Draw distance to 1st mine is D_M
Draw distance to 1st NOMBO is D_O
Draw distance to 1st false alarm is D_F
Draw distance to 1st mine in avoiding direction is D_{M2}
Draw distance to 1st NOMBO in avoiding direction is D_{O2}
Draw distance to 1st false alarm in avoiding direction is D_{F2}
Avoiding Distance is D_{T2}

D_M ~ exp mean $1/(l_M*w)$, D_{M2} ~ exp mean $1/(l_M*w)$
D_O ~ exp mean $1/(l_O*w)$, D_{O2} ~ exp mean $1/(l_O*w)$
D_F ~ exp mean $1/(l_F*w)$, D_{F2} ~ exp mean $1/(l_F*w)$

Do

If Min(D_M, D_O, D_F) > L

Finish

Else if D_M £ Min(D_O, D_F)

Uniform(0, 1) ≤ $P_d(M)$ then draw new D_{M2}, D_{O2}, D_{F2}

Go to first avoiding tactic

Uniform(0, 1) > $P_d(M)$

blowUp

Else If D_O £ Min(D_M, D_F)

Uniform(0, 1) ≤ $P_d(O)$ then draw new D_{M2}, D_{O2}, D_{F2}

Go to first avoiding tactic

Uniform(0, 1) > $P_d(O)$ then draw new D_O (D_O = D_O + new D_O)

Go to Proceeding tactic

Else If D_F £ Min(D_M, D_O) then draw new D_{M2}, D_{O2}, D_{F2}

Go to first avoiding tactic

Until(Finish or Blow up)

Figure 33. Pseudo Code of MOAM Model Simulation (Proceeding Tactic).

Figures 34 and 35 below display the Pseudo Code of the MOAM Model Simulation (first avoiding tactic and second avoiding tactic respectively).

If Min(D_{M2}, D_{O2}, D_{F2}**) >** D_{T2} then draw new D_M, D_O, D_F
($D_{M,O,F}$ = old D_M + new $D_{M,O,F}$)
Go to proceeding tactic
Else if D_{M2} £ **Min(**D_{O2}, D_{F2}**)**
Uniform(0, 1) ≤ $P_d(M)$ then draw new D_{M2}, D_{O2}, D_{F2}
Go to second avoiding tactic
Uniform(0, 1) > $P_d(M)$
blowUp
Else if D_{O2} £ **Min(**D_{M2}, D_{F2}**)**
Uniform(0, 1) ≤ $P_d(O)$ then draw new D_{M2}, D_{O2}, D_{F2}
Go to second avoiding tactic
Uniform(0, 1) > $P_d(O)$ then draw new D_{O2} (D_{O2} = old D_{O2} + new D_{O2})
Go to first avoiding tactic
Else if D_{F2} £ **Min(**D_{M2}, D_{O2}**)** then draw new D_{M2}, D_{O2}, D_{F2}
Go to second avoiding tactic

Figure 34. Pseudo Code of MOAM Model Simulation (First Avoiding Tactic).

If Min(D_{M2}, D_{O2}, D_{F2}**) >** D_{T2} then draw new D_M, D_O, D_F
($D_{M,O,F}$ = old D_M + new $D_{M,O,F}$)
Go to proceeding tactic
Else if D_{M2} £ **Min(**D_{O2}, D_{F2}**)**
Uniform(0, 1) ≤ $P_d(M)$
return, draw new D, and enter again
Uniform(0, 1) > $P_d(M)$
blow up
Else if D_{O2} £ **Min(**D_{M2}, D_{F2}**)**
Uniform(0, 1) ≤ $P_d(O)$
return, draw new D, and enter again
Uniform(0, 1) > $P_d(O)$ then draw new D_{O2} (D_{O2} = old D_{O2} + new D_{O2})
Go to second avoiding tactic
Else if D_{F2} £ **Min(**D_{M2}, D_{O2}**)**
return, draw new D, and enter again

Figure 35. Pseudo Code of MOAM Model Simulation (Second Avoiding Tactic).

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VI. INITIAL ANALYSIS OF THE SIMULATION FOR THE MINEFIELD OBJECT AVOIDANCE MANEUVER MODEL

A. INTRODUCTION

This chapter compares the simulation results with a numerical example using the analytical MOAM model. The fraction of replications resulting in successful minefield transit is compared to the analytical probability of a safe minefield transit. This is computed for the mine-only case, mine + NOMBO case, and mine + NOMBO + false alarm case respectively. The probabilities of a false alarm used in the models appear in Table 48 in Appendix A. All simulation runs have 10,000 replications.

B. PROBABILITY OF SAFE MINEFIELD TRANSIT

The tables below show the output of the analytical SMT model.

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
P_d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
GammaD	0.000	0.001	0.003	0.006	0.011	0.017	0.024	0.033	0.043	0.054	0.067
Beta(D)	0.741	0.760	0.779	0.798	0.818	0.837	0.856	0.875	0.894	0.914	0.933
Theta	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
P(S)	**0.165**	**0.190**	**0.219**	**0.255**	**0.299**	**0.354**	**0.424**	**0.513**	**0.630**	**0.785**	**1.000**
P(F)	0.835	0.810	0.781	0.745	0.701	0.646	0.576	0.487	0.370	0.215	0.000

Table 21. Probability of Safe Minefield Transit (Mine Only Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaO	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
P_d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
GammaD	0.000	0.003	0.010	0.022	0.038	0.058	0.082	0.108	0.138	0.169	0.204
Beta(D)	0.741	0.756	0.768	0.778	0.786	0.792	0.796	0.798	0.799	0.798	0.796
Theta	0.000	0.182	0.333	0.462	0.571	0.667	0.750	0.824	0.889	0.947	1.000
P(S)	**0.165**	**0.181**	**0.201**	**0.226**	**0.257**	**0.297**	**0.350**	**0.422**	**0.526**	**0.692**	**1.000**
P(F)	0.835	0.819	0.799	0.774	0.743	0.703	0.650	0.578	0.474	0.308	0.000

Table 22. Probability of Safe Minefield Transit (Mine and NOMBO Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaO	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
lambdaF	0	0.002	0.005	0.010	0.017	0.029	0.043	0.062	0.091	0.152	1.101
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
P_d	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
GammaD	0.000	0.003	0.011	0.024	0.042	0.068	0.097	0.131	0.179	0.239	0.668
Beta(D)	0.741	0.755	0.767	0.775	0.781	0.781	0.780	0.775	0.757	0.729	0.332
Theta	0.000	0.185	0.340	0.474	0.585	0.687	0.769	0.840	0.905	0.958	1.000
P(S)	**0.165**	**0.181**	**0.200**	**0.223**	**0.252**	**0.285**	**0.331**	**0.394**	**0.474**	**0.611**	**1.000**
P(F)	**0.835**	**0.819**	**0.800**	**0.777**	**0.748**	**0.715**	**0.669**	**0.606**	**0.526**	**0.389**	**0.000**

Table 23. Probability of Safe Minefield Transit (Mine, NOMBO, and False Alarm Case).

lambdaM	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
distance	6	6	6	6	6	6	6	6	6	6	6
width	1	1	1	1	1	1	1	1	1	1	1
Pd(M)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P(S)	**0.165**	**0.198**	**0.237**	**0.284**	**0.340**	**0.407**	**0.487**	**0.583**	**0.698**	**0.835**	**1.000**

*P(S)=exp(-lambdaM*L*w(1-Pd(M)))*

Table 24. Probability of Safe Minefield Transit (Optimistic Case).

The rate of occurrence of mines and NOMBOs used in this analysis is 0.3 mines/mile^2. The minefield distance is 6 miles and the width of the mine actuation is 1 mile. As mentioned previously, the ROC curve model determines the rate of occurrence of false alarms in Table 23 [Appendix A]. The probabilities of mine detection and NOMBO detection are assumed equal, i.e., $P_d(M) = P_d(O) = P_d$. When the ship detects some object in the water, the ship evades the object without classification.

The outputs are compared to an optimistic case which appears in Equation (2). Table 24 above shows the probability of a safe minefield transit for the optimistic case.

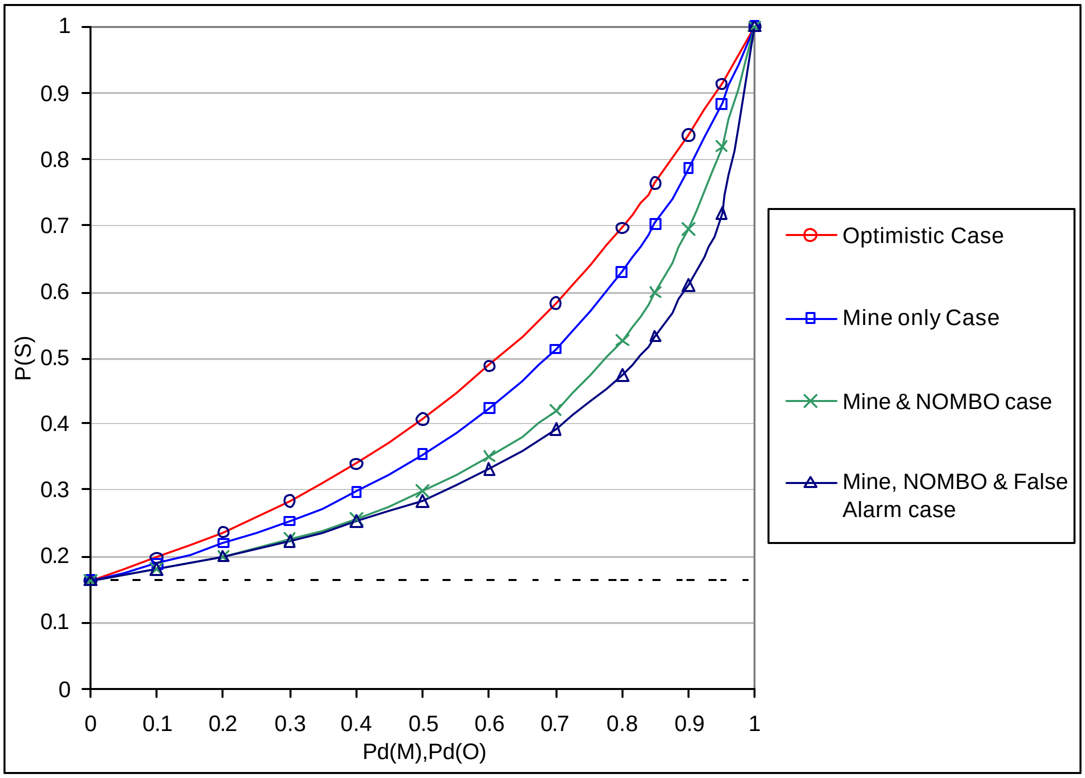

Figure 36. Probability of Safe Minefield Transit in MOAM Model (Analytical Model).

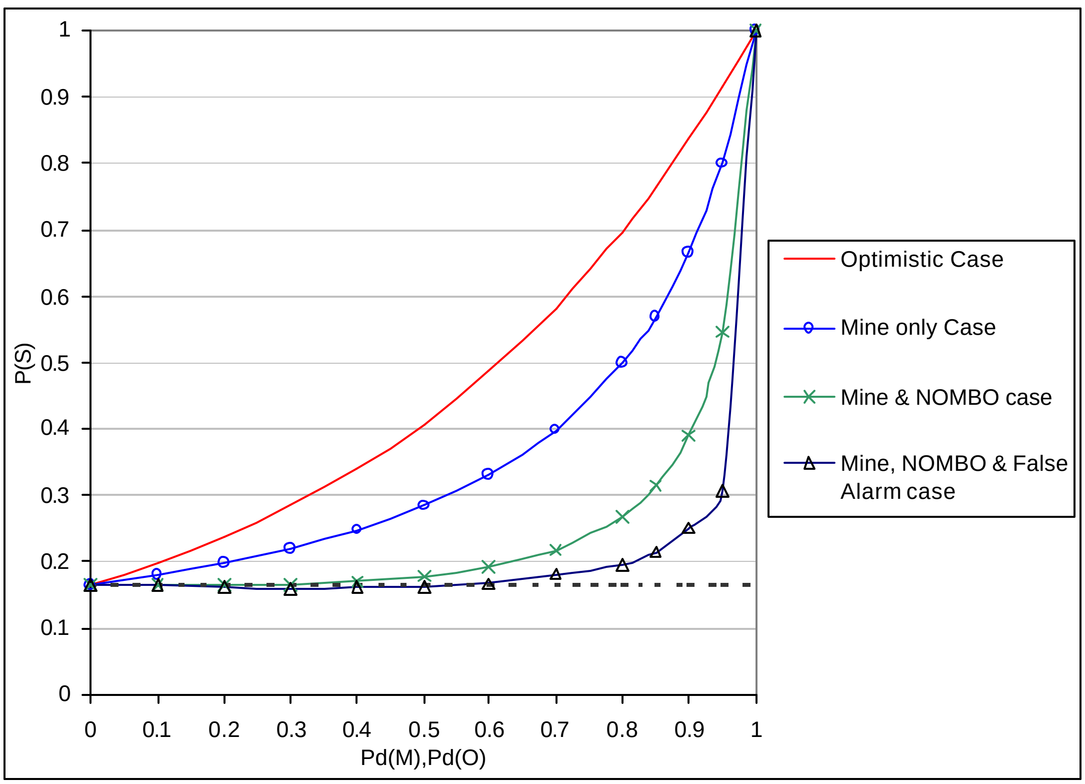

Figure 37. Probability of Safe Minefield Transit in SMT Model (Analytical Model).

Figure 36 shows four cases of the probability of a safe minefield transit in the MOAM model. As can be seen in the above graph, the probability of a safe minefield transit decreases when NOMBOs exist in the minefield and false alarms occur, and increases when the probability of detection increases. Compared to the probability of a safe minefield transit in the SMT model, the probability of safe minefield transit in the MOAM model is increased. Also, when NOMBOs exist in the minefield and false alarms occur, the probability of safe minefield transit in the MOAM model is significantly bigger than that in the SMT model given that the probability of detection is the same. Table 25 below shows the estimates of the probabilities that the ship transits the minefield safely for the MOAM model simulation. Input parameters of the simulation are the same as those of the analytical model. The number of simulation replications is 10,000 for each case. The confidence interval estimates are obtained using a normal approximation, (Devore, 2000)

Pd		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
Mine only	+.95 CI	0.171	0.196	0.224	0.258	0.307	0.363	0.440	0.524	0.635	0.787	1.000
	-.95 CI	0.157	0.181	0.208	0.241	0.289	0.345	0.421	0.504	0.616	0.771	1.000
	Mean	**0.164**	**0.189**	**0.216**	**0.249**	**0.298**	**0.354**	**0.431**	**0.514**	**0.625**	**0.779**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.004	0.000
Analytical Mean		**0.165**	**0.190**	**0.219**	**0.255**	**0.299**	**0.354**	**0.424**	**0.513**	**0.630**	**0.785**	**1.000**
Mine & NOMBO	+.95 CI	0.171	0.188	0.204	0.228	0.271	0.307	0.358	0.422	0.533	0.699	1.000
	-.95 CI	0.157	0.173	0.189	0.212	0.253	0.289	0.339	0.403	0.513	0.681	1.000
	Mean	**0.164**	**0.180**	**0.196**	**0.220**	**0.262**	**0.298**	**0.348**	**0.413**	**0.523**	**0.690**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.000
Analytical Mean		**0.165**	**0.181**	**0.201**	**0.226**	**0.257**	**0.297**	**0.350**	**0.422**	**0.526**	**0.692**	**1.000**
Mine, NOMBO, & false alarm	+.95 CI	0.171	0.188	0.204	0.228	0.257	0.292	0.345	0.410	0.489	0.624	1.000
	-.95 CI	0.157	0.173	0.189	0.212	0.240	0.275	0.326	0.391	0.470	0.605	1.000
	Mean	**0.164**	**0.180**	**0.196**	**0.220**	**0.249**	**0.283**	**0.336**	**0.400**	**0.480**	**0.614**	**1.000**
	Std Err	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.005	0.005	0.000
Analytical Mean		**0.165**	**0.181**	**0.200**	**0.223**	**0.252**	**0.285**	**0.331**	**0.394**	**0.474**	**0.611**	**1.000**

Table 25. Estimate of Probability of Safe Minefield Transit in MOAM Model (Simulation).

Figure 38 below shows the mean probability of a safe minefield transit in the MOAM model and 95% CI graphically. For all cases, the analytical probabilities are within the corresponding 95% confidence intervals obtained from the simulation model.

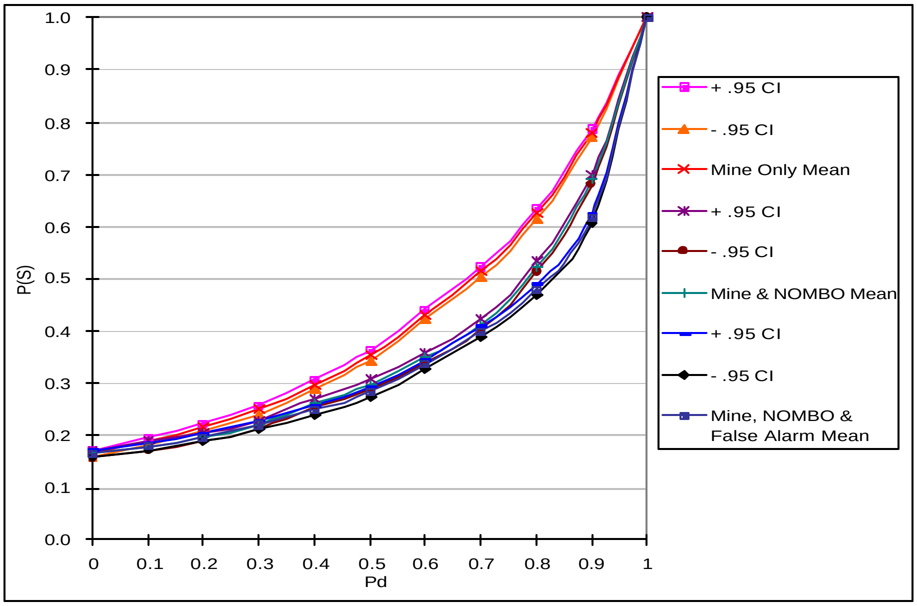

Figure 38. Estimate of Probability of Safe Minefield Transit in MOAM model and 95% CI.

C. OTHER SIMULATION RESULTS

1. Conditional Mean Distance Traveled Given Unsuccessful Transit

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
Mine only	+.95 CI	2.173	2.306	2.455	2.626	2.784	3.091	3.326	3.677	4.098	4.661	5.472
	- .95 CI	2.104	2.233	2.376	2.541	2.690	2.982	3.196	3.520	3.887	4.357	4.379
	Mean	**2.138**	**2.269**	**2.416**	**2.584**	**2.737**	**3.036**	**3.261**	**3.599**	**3.992**	**4.509**	**4.926**
	Std Err	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.018
Mine & NOMBO	+.95 CI	2.173	2.387	2.589	2.907	3.193	3.678	4.489	5.444	7.034	9.385	14.75
	- .95 CI	2.104	2.312	2.504	2.807	3.075	3.531	4.291	5.185	6.651	8.727	12.36
	Mean	**2.138**	**2.349**	**2.547**	**2.857**	**3.134**	**3.604**	**4.390**	**5.314**	**6.842**	**9.056**	**13.56**
	Std Err	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.003	0.030
Mine, NOMBO, & false alarm	+.95 CI	2.173	2.355	2.584	2.855	3.323	3.919	4.714	6.000	8.757	14.33	284.9
	- .95 CI	2.104	2.280	2.500	2.757	3.198	3.760	4.501	5.699	8.273	13.41	269.2
	Mean	**2.138**	**2.318**	**2.542**	**2.806**	**3.261**	**3.840**	**4.608**	**5.850**	**8.515**	**13.87**	**277.1**
	Std Err	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.004	0.057

Table 26. Conditional Mean Distance Traveled Given Unsuccessful Transit (Simulation).

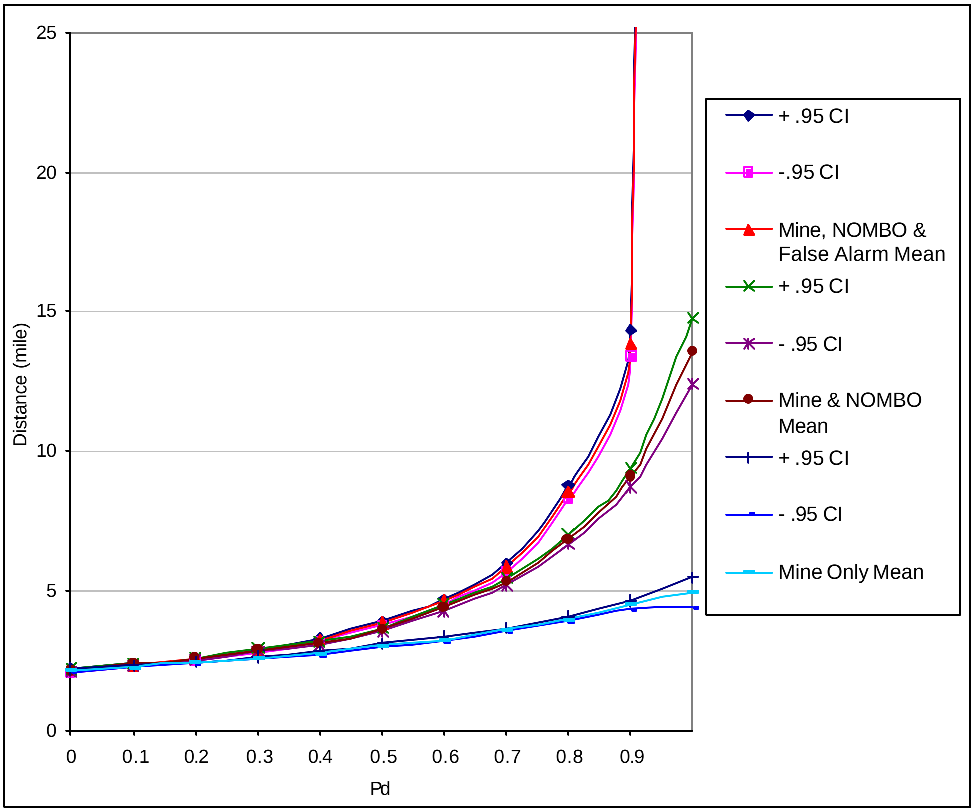

Figure 39. Conditional Mean Distance Traveled Given Unsuccessful Transit and 95% CI.

Table 26 and Figure 39 above show the conditional mean distance traveled given an unsuccessful minefield transit as the probability of detection increases. In this case, the results do not exist when the probability of detection is 1.0. Therefore, the probability of detection 0.99 is used instead of 1.0 to obtain the extreme results. The conditional mean distance traveled given unsuccessful minefield transit increases as the probability of detection increases and it increases quickly when NOMBOs and false alarms exist.

Figures 40 and 41 are the histograms of the distance traveled given unsuccessful transit, when mines and NOMBOs exist in the field and no false alarms occur. Two cases (low and high rates of occurrence of NOMBOs in the minefield) are compared to study how the NOMBO acts on the minefield travel distance given unsuccessful transit. All the assumptions are same as those used in this chapter except that the width of mine actuation is 0.5 miles. The low (respectively high) rate of occurrence of the NOMBOs is 0.3 (respectively 1.5).

It is noted that the distances captured in the simulation are optimistic. At each maneuver, the distance calculated in the simulation doesn't count the offset *w*/2 distance units left or right of the original track until the transiting ship enters a non-overlapping area of the minefield. Nor do the simulation models count actual distance a real ship may move while executing a turn. These details could be added.

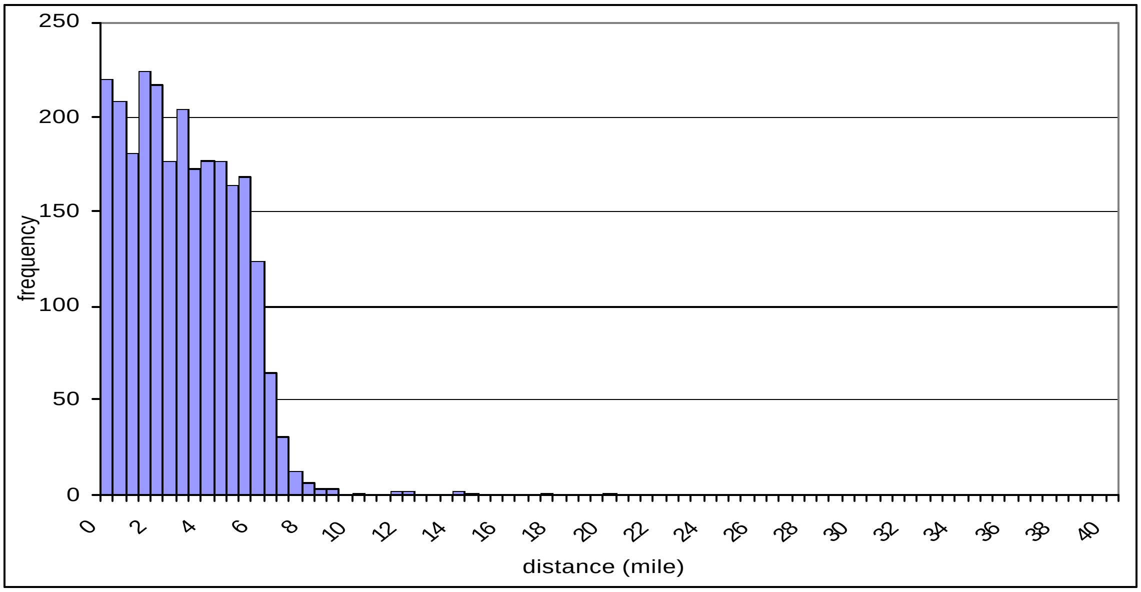

Figure 40. Histogram of Distance Traveled Given Unsuccessful Transit, L=6, w=0.5, l_M=0.3, l_O=0.3, l_F=0.0, P_d=0.7.

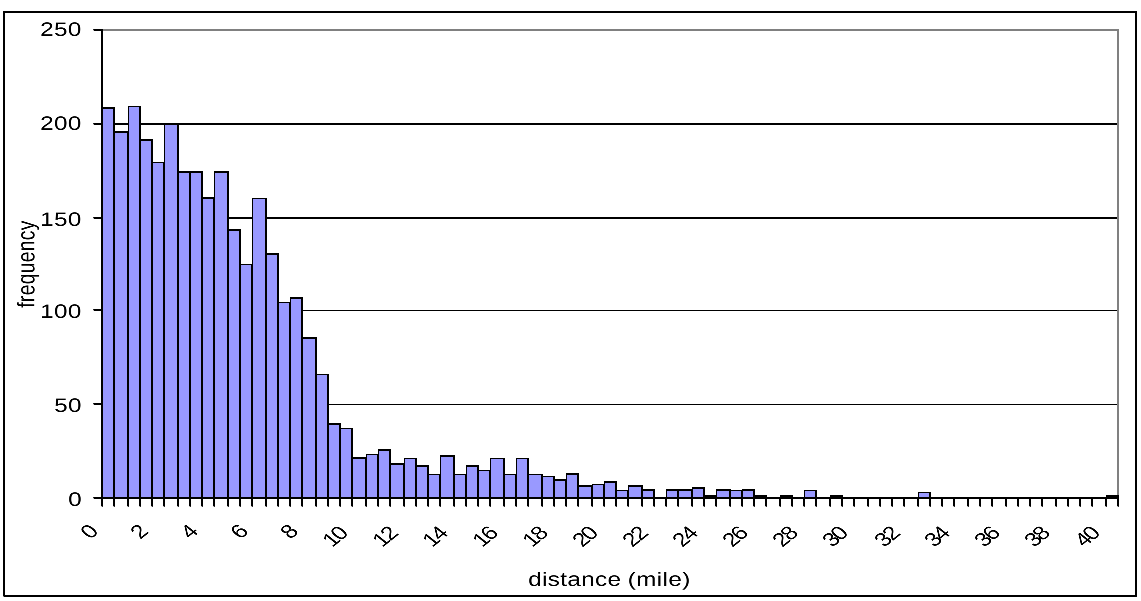

Figure 41. Histogram of Distance Traveled Given Unsuccessful Transit, L=6, w=0.5, l_M=0.3, l_O=1.5, l_F=0.0, P_d=0.7.

The total number of observations displayed in Figure 40 is 2543 and in Figure 41 is 3278. The mean and maximum distances traveled as displayed in Figure 40 are 3.26 miles and 20.25 miles, and in Figure 41 are 5.54 miles and 42.44 miles, respectively. This demonstrates that, when the rate of occurrence of NOMBOs increases in the minefield, the mean and maximum distances traveled also increase.

2. Conditional Mean Distance Traveled Given Successful Transit

Table 27 and Figure 42 below show the conditional mean distance traveled given a successful transit, as the probability of detection increases. The mean distance traveled given successful transit increases as the probability of detection increases. According to the ROC curve model, when the probability of detecting a mine ($P_d(M)$) is 1, the probability of a false alarm (P_f) is almost 1. As a result, the rate of a false alarm (l_F) becomes 1.101, which makes the total distance extremely long.

Pd		**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
Mine only	+.95 CI	6.000	6.166	6.376	6.517	6.750	6.988	7.258	7.622	8.013	8.592	9.260
	- .95 CI	6.000	6.131	6.319	6.453	6.672	6.897	7.158	7.507	7.886	8.444	9.100
	Mean	**6.000**	**6.149**	**6.347**	**6.485**	**6.711**	**6.942**	**7.208**	**7.564**	**7.949**	**8.518**	**9.180**
	Std Err	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Mine & NOMBO	+.95 CI	6.000	6.311	6.657	7.097	7.569	8.234	9.098	10.266	11.679	14.275	19.421
	- .95 CI	6.000	6.258	6.579	6.985	7.423	8.048	8.857	9.977	11.342	13.851	18.864
	Mean	**6.000**	**6.284**	**6.618**	**7.041**	**7.496**	**8.141**	**8.977**	**10.122**	**11.511**	**14.063**	**19.143**
	Std Err	0.000	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
Mine, NOMBO, & false alarm	+.95 CI	6.000	6.319	6.680	7.160	7.740	8.581	9.689	11.302	14.004	20.271	596.02
	- .95 CI	6.000	6.269	6.600	7.035	7.569	8.360	9.419	10.937	13.521	19.532	573.15
	Mean	**6.000**	**6.294**	**6.640**	**7.097**	**7.654**	**8.471**	**9.554**	**11.120**	**13.763**	**19.902**	**584.59**
	Std Err	0.000	0.000	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.058

Table 27. Conditional Mean Distance Traveled Given Successful Transit (Simulation).

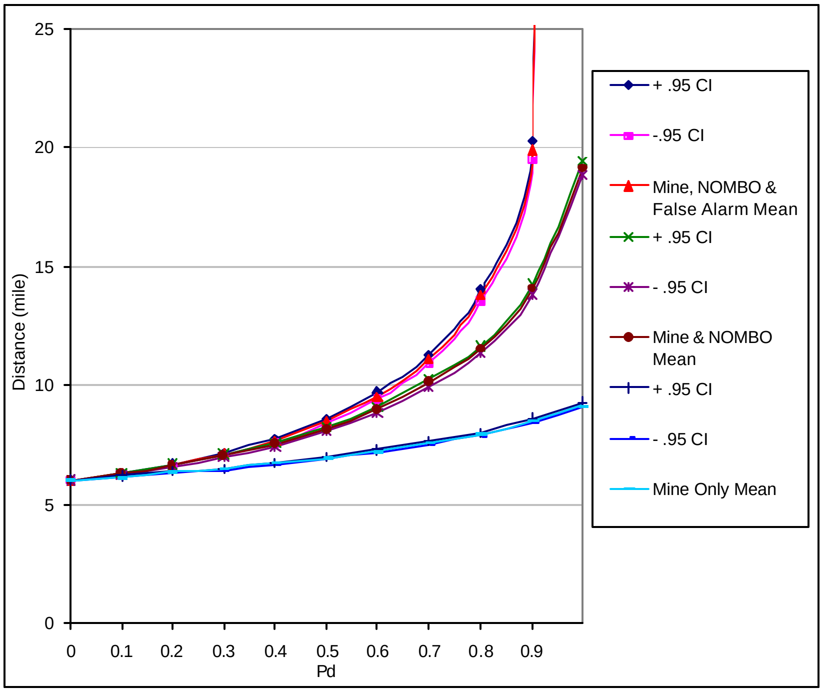

Figure 42. Conditional Mean Distance Traveled Given Successful Transit and 95% CI.

Figures 43 and 44 below display histograms of the distance traveled given successful transit, when mines and NOMBOs exist in the field and no false alarms occur, and the distance traveled is greater than the distance of the minefield. Two cases (low and high rate of occurrence of NOMBOs in the minefield) are compared to study how the rate of occurrence of NOMBOs influences the minefield travel distance given successful transit. The low (respectively high) rate of occurrence of the NOMBOs is 0.3 (respectively 1.5).

When $l_o = 0.3$, the fraction of replications in which the distance traveled given successful transit equals to the distance of the minefield is 2,143/7,457 = 0.2874. When $l_o = 1.5$, the fraction of replications in which the distance is equal to the distance of the minefield is 176/6,722 = 0.0262. The conditional distribution of the distance traveled given successful transit is not that of a continuous random variable. The replications that the distance traveled given successful transit equals to the distance of the minefield are truncated from the original data. The resulting histograms for the remaining distance data appear in Figures 43 and 44. The total number of observations displayed in Figure 43 is 5314 and in Figure 44 is 6546. The mean and maximum distances traveled as displayed

in Figure 43 are 7.04 miles and 21.36 miles, and in Figure 44 are 9.96 miles and 49.63 miles, respectively. This demonstrates that, when the rate of occurrence of NOMBOs in the minefield increases, the mean and maximum distances traveled also increase. In the MOAM model, when the rate of occurrence of NOMBO increases, the mean increases and part of the histogram distribution begins to exhibit a bell-like shape.

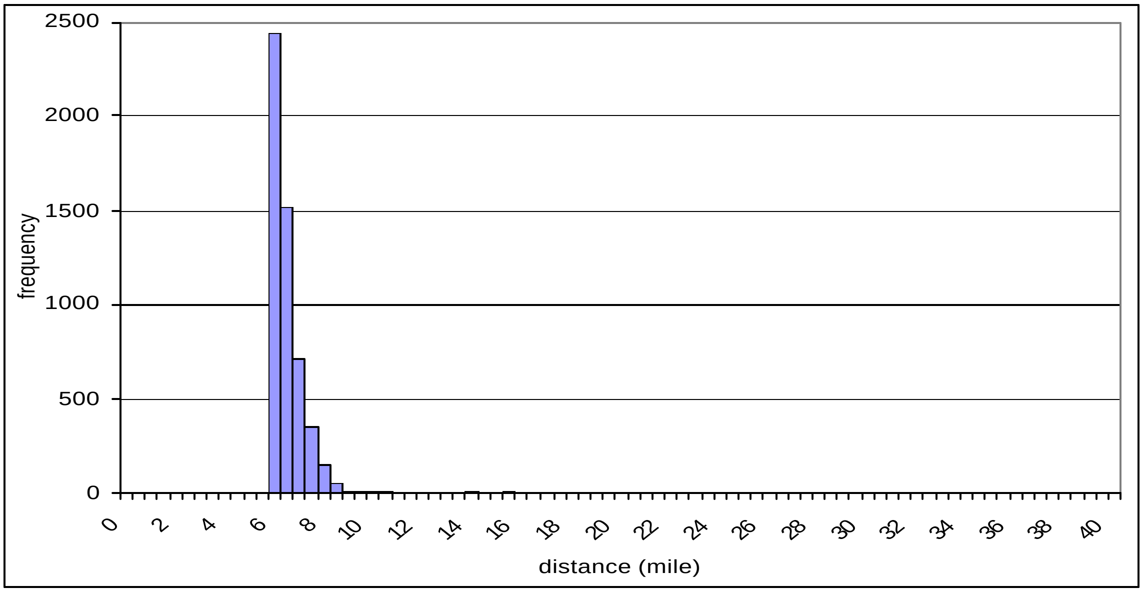

Figure 43. Histogram of Distance Traveled Given Successful Transit, L=6, w=0.5, l_M =0.3, l_O =0.3, l_F =0.0, P_d=0.7.

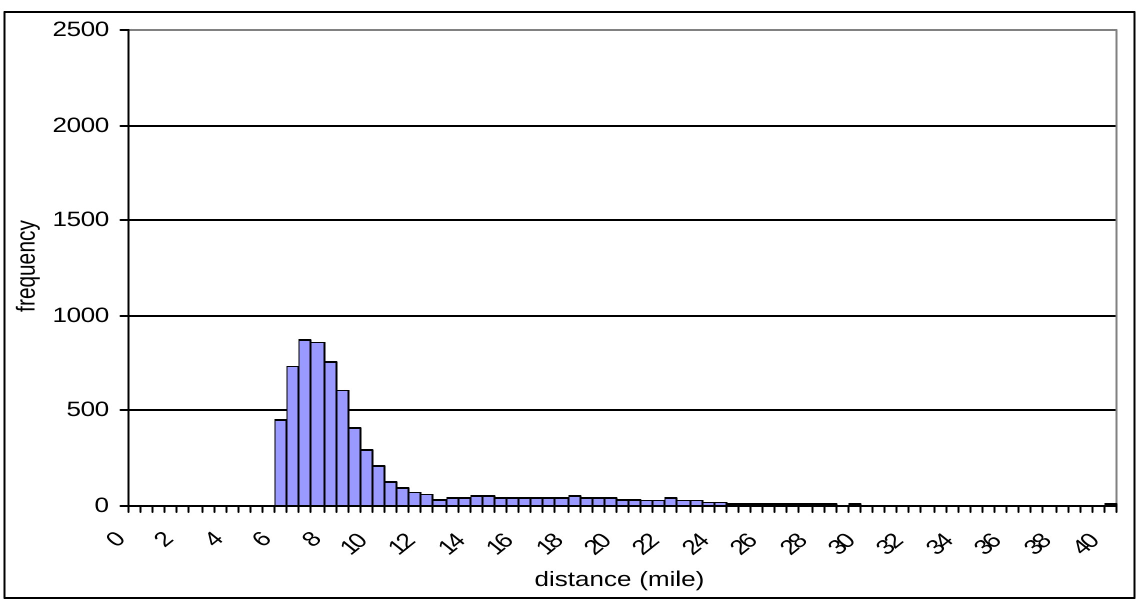

Figure 44. Histogram of Distance Traveled Given Successful Transit, L=6, w=0.5, l_M =0.3, l_O =1.5, l_F =0.0, P_d=0.7.

3. Mean Distance Traveled

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mine only	+.95 CI	2.811	3.043	3.309	3.602	3.971	4.472	5.018	5.698	6.533	7.706	9.260
	- .95 CI	2.730	2.958	3.219	3.509	3.872	4.366	4.903	5.574	6.398	7.557	9.100
	Mean	**2.771**	**3.001**	**3.264**	**3.556**	**3.922**	**4.419**	**4.961**	**5.636**	**6.465**	**7.632**	**9.180**
	Std Err	0.0002	0.0002	0.0002	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004
Mine & NOMBO Object	+.95 CI	2.811	3.102	3.393	3.832	4.337	5.029	6.076	7.406	9.418	12.696	19.421
	- .95 CI	2.730	3.015	3.299	3.725	4.216	4.887	5.899	7.191	9.148	12.328	18.864
	Mean	**2.771**	**3.059**	**3.346**	**3.778**	**4.277**	**4.958**	**5.987**	**7.298**	**9.283**	**12.512**	**19.143**
	Std Err	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0004	0.0005	0.0007	0.0009	0.0014
Mine, NOMBO object, & false alarm	+.95 CI	2.811	3.077	3.394	3.802	4.417	5.228	6.364	8.085	11.210	17.868	596.02
	- .95 CI	2.730	2.990	3.299	3.695	4.290	5.075	6.173	7.832	10.852	17.279	573.15
	Mean	**2.771**	**3.034**	**3.346**	**3.749**	**4.353**	**5.152**	**6.268**	**7.959**	**11.031**	**17.573**	**584.59**
	Std Err	0.0002	0.0002	0.0002	0.0003	0.0003	0.0004	0.0005	0.0006	0.0009	0.0015	0.0583

Table 28. Mean Distance Traveled (Simulation).

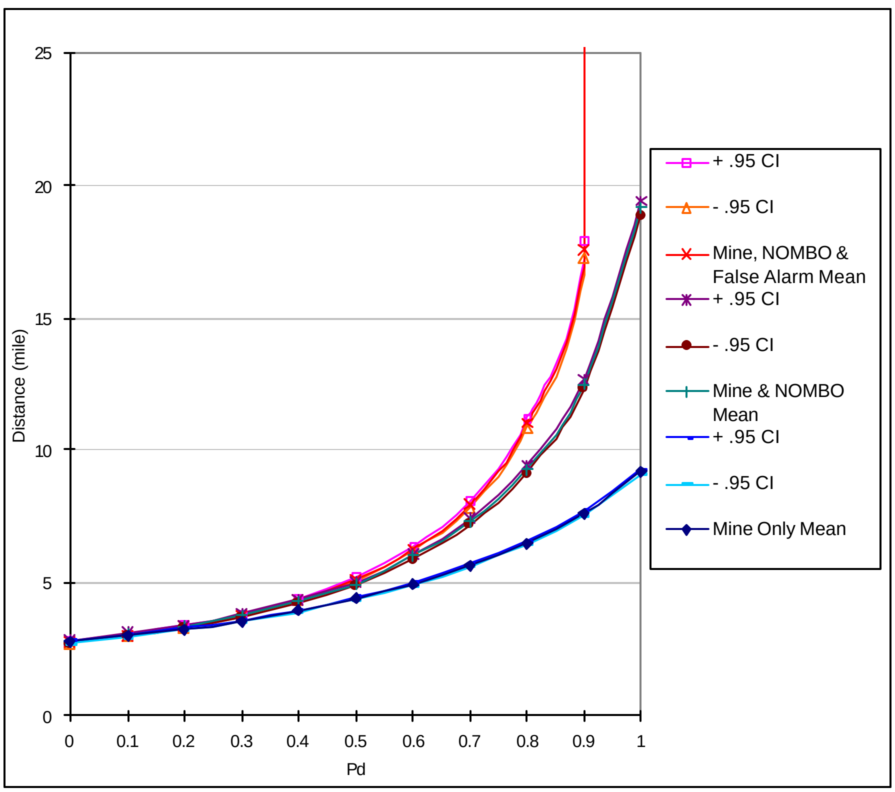

Figure 45. Mean Distance Traveled and 95% CI.

Table 28 and Figure 45 above shows the mean distance traveled. This mean distance traveled contains not only the distance traveled given successful transit but also the distance traveled given unsuccessful transit.

4. Mean Number of Retracings

Table 29 and Figure 46 below show the mean number of retracings (returns to the entry to the minefield) according to the probability of detection. The mean number of retracings increases as the probability of detection increases.

Pd		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mine only	+.95 CI	0.0000	0.0000	0.0009	0.0016	0.0077	0.0145	0.0217	0.0363	0.0566	0.0899	0.1376
	- .95 CI	0.0000	0.0000	0.0001	0.0004	0.0045	0.0101	0.0163	0.0291	0.0474	0.0783	0.1226
	Mean	**0.0000**	**0.0000**	**0.0005**	**0.0010**	**0.0061**	**0.0123**	**0.0190**	**0.0327**	**0.0520**	**0.0841**	**0.1301**
	Std Err	0.0000	0.0000	0.0002	0.0003	0.0008	0.0011	0.0014	0.0018	0.0023	0.0030	0.0038
Mine & NOMBO Object	+.95 CI	0.0000	0.0009	0.0048	0.0148	0.0316	0.0673	0.1254	0.2099	0.3512	0.5923	1.1088
	- .95 CI	0.0000	0.0001	0.0024	0.0104	0.0250	0.0573	0.1110	0.1909	0.3250	0.5551	1.0500
	Mean	**0.0000**	**0.0005**	**0.0036**	**0.0126**	**0.0283**	**0.0623**	**0.1182**	**0.2004**	**0.3381**	**0.5737**	**1.0794**
	Std Err	0.0000	0.0002	0.0006	0.0011	0.0017	0.0026	0.0037	0.0048	0.0067	0.0095	0.0150
Mine, NOMBO object, & false alarm	+.95 CI	0.0000	0.0009	0.0049	0.0162	0.0420	0.0859	0.1649	0.2984	0.5756	1.2525	118.59
	- .95 CI	0.0000	0.0001	0.0025	0.0116	0.0342	0.0743	0.1485	0.2744	0.5390	1.1873	113.96
	Mean	**0.0000**	**0.0005**	**0.0037**	**0.0139**	**0.0381**	**0.0801**	**0.1567**	**0.2864**	**0.5573**	**1.2199**	**116.28**
	Std Err	0.0000	0.0002	0.0006	0.0012	0.0020	0.0030	0.0042	0.0061	0.0094	0.0166	1.1806

Table 29. Mean Number of Retracings (Simulation).

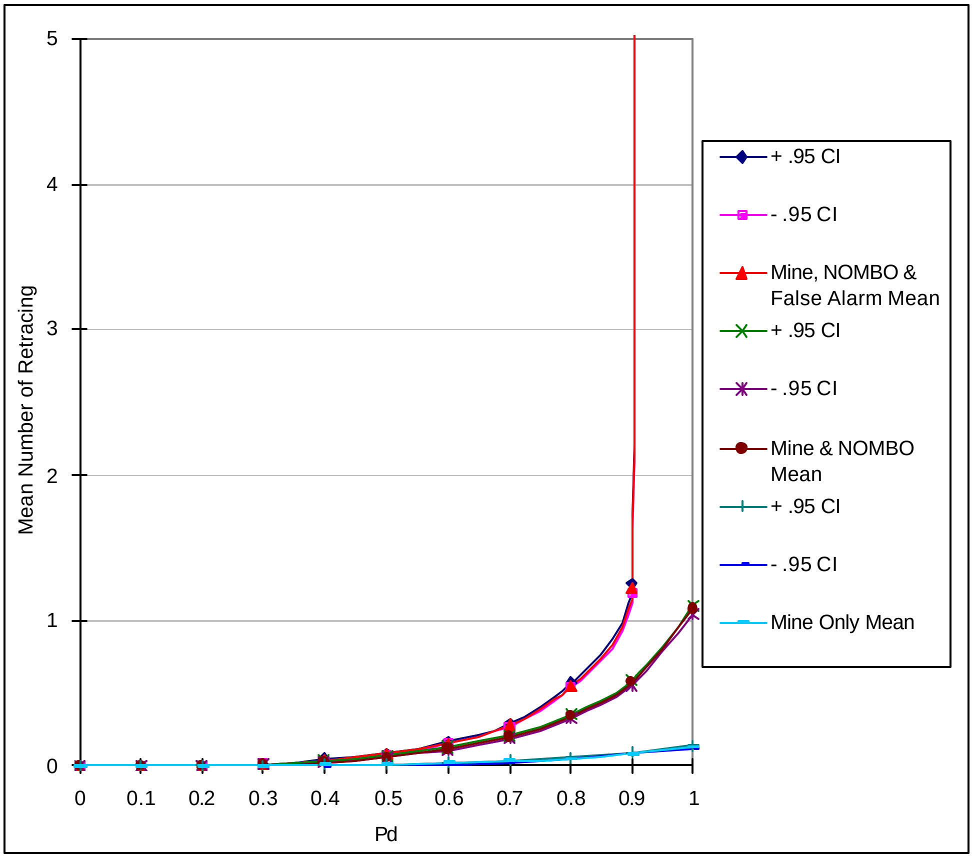

Figure 46. Mean Number of Retracings in the Minefield and 95% CI.

D. DISCUSSION

Using the probability of a safe minefield transit (as a function of the probability of mine and NOMBO detection) as the MOE, the simulation output compares well to the analytical SMT model. The analytical calculation results are within the 95% confidence interval of the simulation outputs with the same input. This suggests the simulation is consistent with the analytical SMT model for these parameters.

Additionally, comparing the MOAM model to the SMT model, the more sophisticated maneuver of the MOAM model results in a larger probability of survival. The probability of a safe minefield transit of the MOAM model is significantly higher, when the probability of detection is the same, and the mean distance traveled given successful transit and the mean number of retracing in the minefield is significantly lower. Thus, the representation of a more realistic maneuvering tactic is important in the evaluation of the mine avoidance tactic.

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VII. EFFECT OF THE NOMBO AND FALSE ALARM ON THE SAFE MINEFIELD TRANSIT IN MOAM MODEL

A. INTRODUCTION

This chapter explores the effects of the NOMBOs and false alarms on the probability of safe minefield transit in the MOAM model. This is accomplished by varying the rate of the occurrence of the NOMBOs $\left(l_o\right)$ and the detection index (*d*) respectively and is compared with a no sensor case when a ship transits the minefield along a direct, straight line without a sensor. If a probability of a safe minefield transit (*P(S)*), when using a sensor, is less than or equal to that in a no sensor case, there is no benefit to the ship using a sensor to transit the minefield. The avoiding distance $\left(\bar{h}\right)$ is the same as the mine actuation width of the ship (*w*) in this model. The rate of occurrence of NOMBOs ranges from 0 to 1.0 in increments of 0.1. The detection index ranges from 0 to 10.0. On the intervals 0 to 1.0, the increments are 0.2. On the intervals 2.0 to 10.0, the increments are 2. The probabilities of a false alarm used in the models appear in Table 48 in Appendix A. The Measure of Effectiveness under investigation is the probability of a safe minefield transit, and if and how the change of rate of the occurrence of NOMBOs $\left(l_o\right)$ or detection index (*d*) affects this probability. The analytical model in equation (3) is used to obtain the results in this section. However, the results could also have been obtained using the simulation.

B. INPUT PARAMETERS

The table shows the input parameters used in the analytical models.

Environment	Minefield Distance (L)	Mine Actuation width of ship (w), Avoiding Dist($\bar{h}$)	Rate of Mine (l_M)	Rate of NOMBO (l_0)	Detection Index (d)
Mine Only	6	.5, 1.0	0.1 ~ 1.0	0	-
Mine & NOMBO	6	1.0	0.3, 0.5, 0.7	0.0 ~ 1.0	-
Mine & False Alarm	6	1.0	0.3, 0.5, 0.7	0	0.0 ~ 10.0
Mine, NOMBO & False Alarm	6	1.0	0.3, 0.5, 0.7	.6	0.0 ~ 10.0

Table 30. Input Parameters for Each Environment.

Input parameters of the MOAM model are different from those of SMT model, because, if the latter is used in the MOAM model, the case that the probability of a safe minefield transit with a sensor is less than or equal to that with no sensor never happens.

C. OUTPUTS

1. Mine Only Case

lambdaM parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.741	0.763	0.786	0.809	0.834	0.859	0.885	0.912	0.941	0.970	1.000
0.2	0.549	0.581	0.616	0.653	0.693	0.735	0.781	0.830	0.883	0.939	1.000
0.3	0.407	0.442	0.482	0.525	0.573	0.627	0.686	0.752	0.826	0.908	1.000
0.4	0.301	0.336	0.376	0.421	0.473	0.532	0.601	0.680	0.771	0.877	1.000
0.5	0.223	0.255	0.293	0.337	0.389	0.450	0.524	0.612	0.717	0.845	1.000
0.6	0.165	0.193	0.227	0.268	0.318	0.380	0.455	0.548	0.665	0.812	1.000
0.7	0.122	0.146	0.176	0.213	0.260	0.319	0.393	0.490	0.615	0.779	1.000
0.8	0.091	0.111	0.136	0.169	0.211	0.266	0.339	0.435	0.566	0.746	1.000
0.9	0.067	0.084	0.105	0.134	0.171	0.222	0.290	0.385	0.519	0.712	1.000
1.0	0.050	0.063	0.081	0.105	0.138	0.184	0.248	0.339	0.474	0.677	1.000

Table 31. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_O=0.0$.

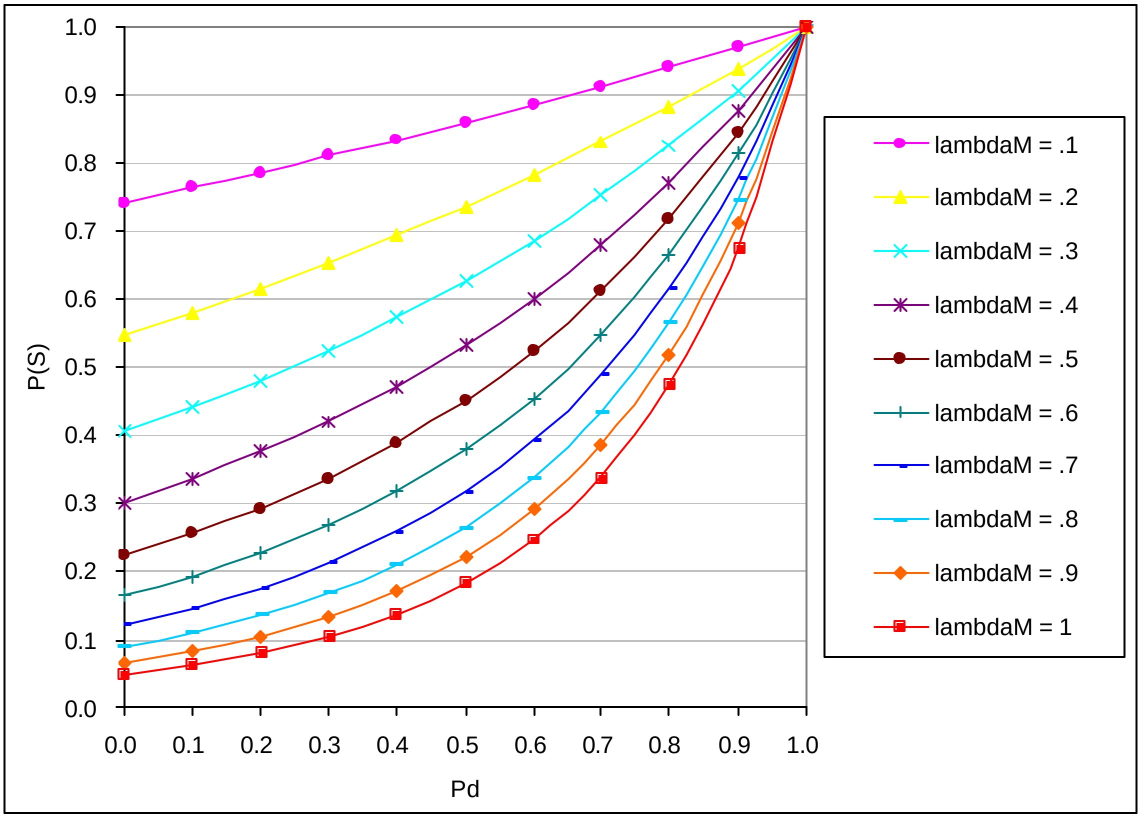

Figure 47. Probability of Safe Minefield Transit, $L=6$, $w=0.5$, $l_o=0.0$.

lambdaM parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.1	0.549	0.580	0.613	0.649	0.688	0.730	0.775	0.825	0.878	0.936	1.000
0.2	0.301	0.333	0.369	0.411	0.460	0.517	0.583	0.661	0.754	0.865	1.000
0.3	0.165	0.190	0.219	0.255	0.299	0.354	0.424	0.513	0.630	0.785	1.000
0.4	0.091	0.107	0.128	0.155	0.189	0.236	0.298	0.384	0.509	0.696	1.000
0.5	0.050	0.060	0.074	0.092	0.117	0.152	0.202	0.277	0.395	0.598	1.000
0.6	0.027	0.034	0.042	0.054	0.071	0.095	0.132	0.191	0.293	0.493	1.000
0.7	0.015	0.019	0.024	0.031	0.042	0.058	0.083	0.127	0.207	0.386	1.000
0.8	0.008	0.010	0.013	0.018	0.024	0.034	0.051	0.081	0.139	0.286	1.000
0.9	0.005	0.006	0.007	0.010	0.014	0.020	0.030	0.049	0.089	0.199	1.000
1.0	0.002	0.003	0.004	0.006	0.008	0.011	0.018	0.029	0.055	0.131	1.000

Table 32. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_o=0.0$.

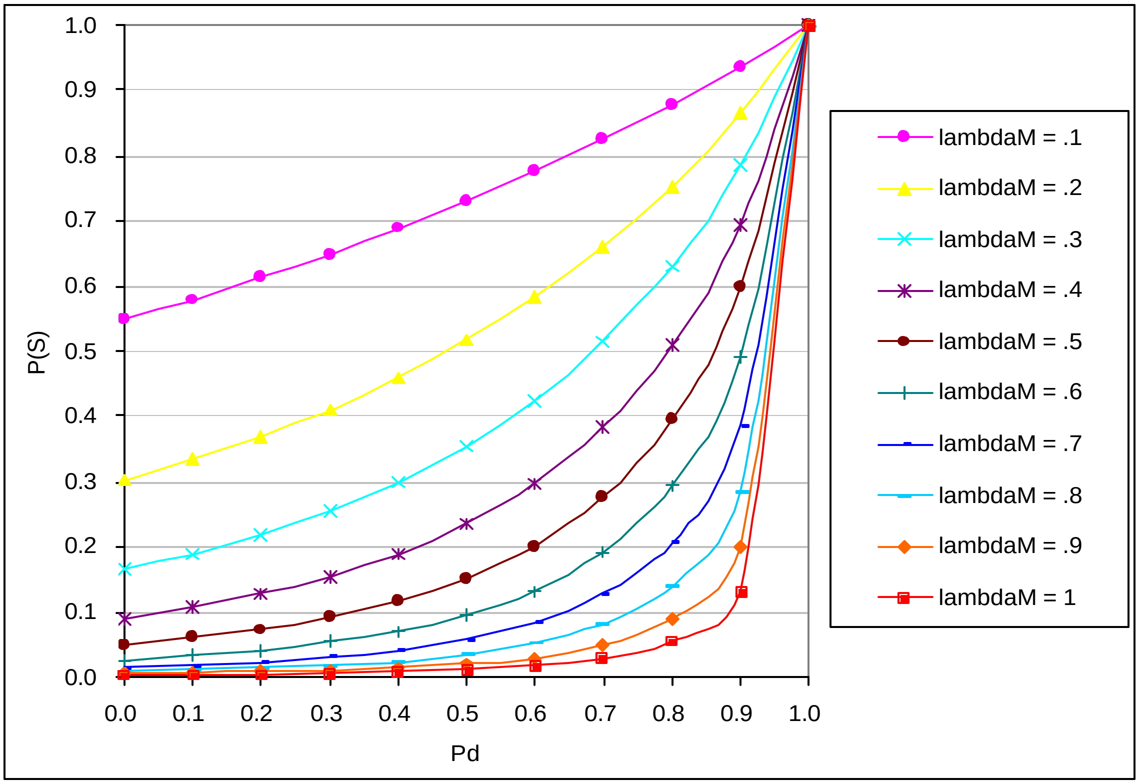

Figure 48. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$.

Tables 31 and 32 and Figures 47 and 48 above are generated from the results of those simulation runs. The probabilities of a safe minefield transit are plotted. Tables 31 and 32 show the statistics for various l_M. The probabilities clearly show a pattern. When the probability of detection increases and the rate of occurrence of deployed mines decreases, the probability of a safe minefield transit increases. In addition, there is no probability of a safe minefield transit with a sensor that is less than that of a no sensor case, which means that, whenever a ship uses a sensor, the probability of a safe minefield transit never decreases. Thus, it is beneficial to the ship to use a sensor, even when the probability of detection of the sensor is low while transiting the minefield in this case in which there are no NOMBOs and no false alarms.

2. Mine and NOMBO Case

The tables below show the probability of safe minefield transit, when the rate of occurrence of mines is 0.3, 0.5, and 0.7 respectively. The rate of occurrence of NOMBOs ranges from 0.0 to 1.0 in increments of 0.1. There are no false alarms. The

effect of the rate of occurrence of NOMBOs on the probability of a safe minefield transit is shown as the rate of the occurrence of mines in the field increases. The case with a probability of detection equal to 0 is that a ship does not use a sensor and transits the minefield in a straight line.

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	0.190	0.219	0.255	0.299	0.354	0.424	0.513	0.630	0.785	1.000
0.1	0.165	0.187	0.213	0.245	0.285	0.336	0.401	0.486	0.600	0.761	1.000
0.2	0.165	0.184	0.207	0.236	0.271	0.317	0.376	0.455	0.566	0.731	1.000
0.3	0.165	0.181	0.201	0.226	0.257	0.297	0.350	0.422	0.526	0.692	1.000
0.4	0.165	0.179	0.195	0.216	0.243	0.277	0.322	0.385	0.481	0.645	1.000
0.5	0.165	0.176	0.189	0.207	0.228	0.256	0.293	0.346	0.430	0.587	1.000
0.6	0.165	0.173	0.184	0.197	0.213	0.234	0.263	0.305	0.375	0.519	1.000
0.7	0.165	0.171	0.178	0.187	0.198	0.213	0.233	0.263	0.317	0.441	1.000
0.8	0.165	0.168	0.172	0.177	0.184	0.192	0.203	0.222	0.260	0.358	1.000
0.9	0.165	**0.165**	0.166	0.168	0.169	0.171	0.174	0.183	0.205	0.277	1.000
1.0	0.165	**0.163**	**0.160**	**0.158**	**0.154**	**0.150**	**0.147**	**0.147**	**0.156**	0.202	1.000

Table 33. Probability of Safe Minefield Transit, *L=6, w=1.0,* l_M *=0.3.*

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.060	0.074	0.092	0.117	0.152	0.202	0.277	0.395	0.598	1.000
0.1	0.050	0.059	0.071	0.087	0.108	0.138	0.181	0.247	0.354	0.550	1.000
0.2	0.050	0.057	0.068	0.081	0.099	0.125	0.161	0.217	0.311	0.495	1.000
0.3	0.050	0.056	0.065	0.076	0.091	0.112	0.141	0.187	0.267	0.432	1.000
0.4	0.050	0.055	0.062	0.071	0.083	0.099	0.122	0.159	0.222	0.364	1.000
0.5	0.050	0.054	0.059	0.066	0.075	0.087	0.104	0.131	0.179	0.294	1.000
0.6	0.050	0.052	0.056	0.061	0.067	0.076	0.088	0.106	0.140	0.226	1.000
0.7	0.050	0.051	0.053	0.056	0.060	0.065	0.072	0.084	0.106	0.166	1.000
0.8	0.050	0.050	0.051	0.052	0.054	0.056	0.059	0.065	0.077	0.116	1.000
0.9	0.050	**0.049**	**0.048**	**0.048**	**0.047**	**0.047**	**0.047**	**0.049**	0.055	0.077	1.000
1.0	0.050	**0.048**	**0.046**	**0.044**	**0.042**	**0.039**	**0.037**	**0.036**	**0.038**	**0.050**	1.000

Table 34. Probability of Safe Minefield Transit, *L=6, w=1.0,* l_M *=0.5.*

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.015	0.019	0.024	0.031	0.042	0.058	0.083	0.127	0.207	0.386	1.000
0.1	0.015	0.018	0.022	0.029	0.037	0.050	0.071	0.106	0.172	0.327	1.000
0.2	0.015	0.018	0.021	0.026	0.033	0.044	0.060	0.087	0.138	0.266	1.000
0.3	0.015	0.017	0.020	0.024	0.029	0.037	0.050	0.070	0.108	0.207	1.000
0.4	0.015	0.017	0.019	0.022	0.026	0.032	0.041	0.055	0.082	0.154	1.000
0.5	0.015	0.016	0.018	0.020	0.023	0.027	0.033	0.042	0.060	0.110	1.000
0.6	0.015	0.016	0.017	0.018	0.020	0.022	0.026	0.032	0.043	0.075	1.000
0.7	0.015	0.015	0.016	0.016	0.017	0.019	0.020	0.024	0.030	0.049	1.000
0.8	0.015	**0.015**	**0.015**	**0.015**	**0.015**	0.015	0.016	0.017	0.020	0.031	1.000
0.9	0.015	**0.014**	**0.014**	**0.013**	**0.013**	**0.012**	**0.012**	**0.012**	**0.013**	0.019	1.000
1.0	0.015	**0.014**	**0.013**	**0.012**	**0.011**	**0.010**	**0.009**	**0.009**	**0.009**	**0.011**	1.000

Table 35. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_M=0.7$.

The colored boxes in the above tables indicate that the probability of a safe minefield transit is less than or equal to that of a no sensor case.

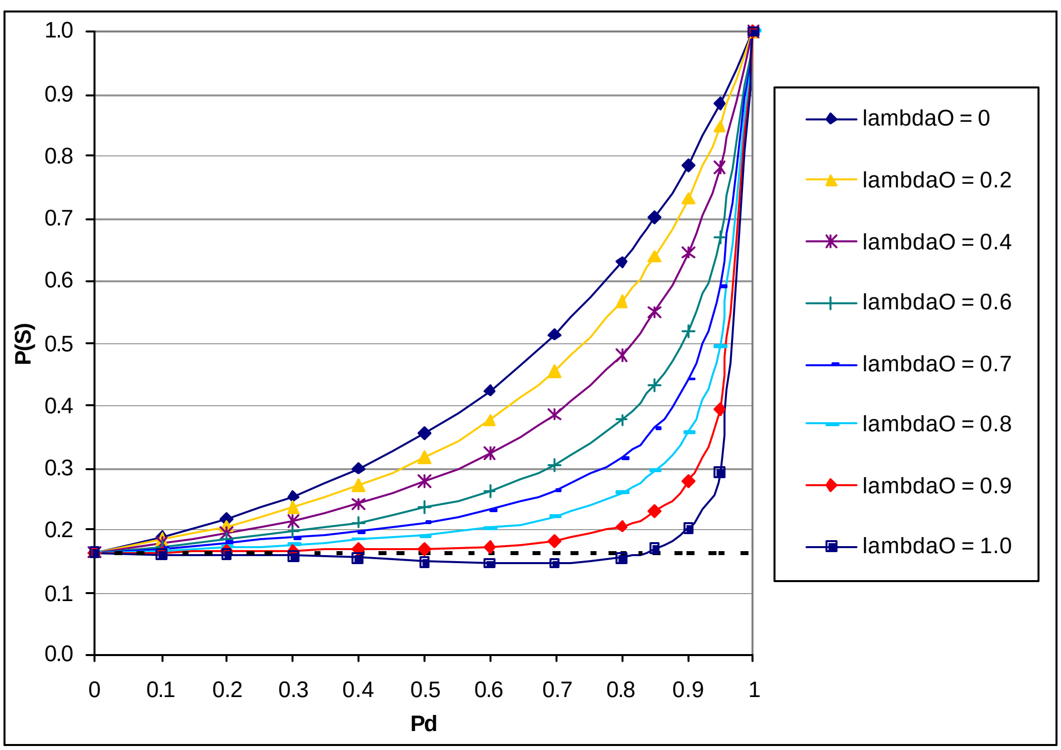

Figure 49. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_M=0.3$.

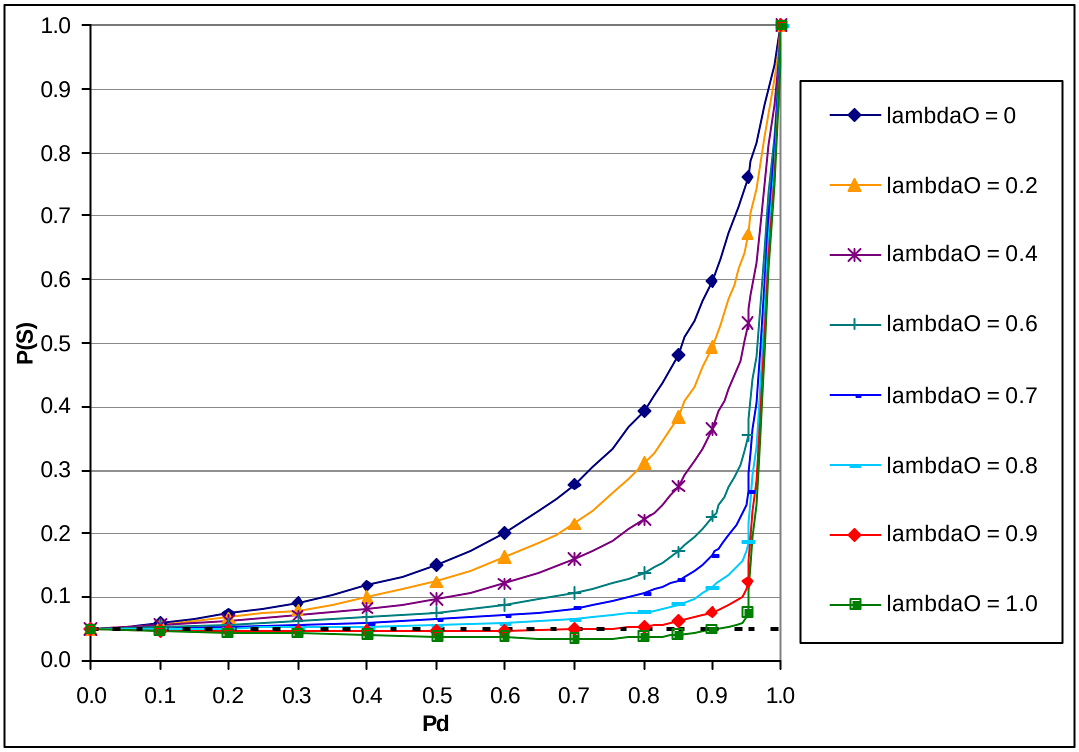

Figure 50. Probability of Safe Minefield Transit, *L=6, w=1.0,* l_M *=0.5*.

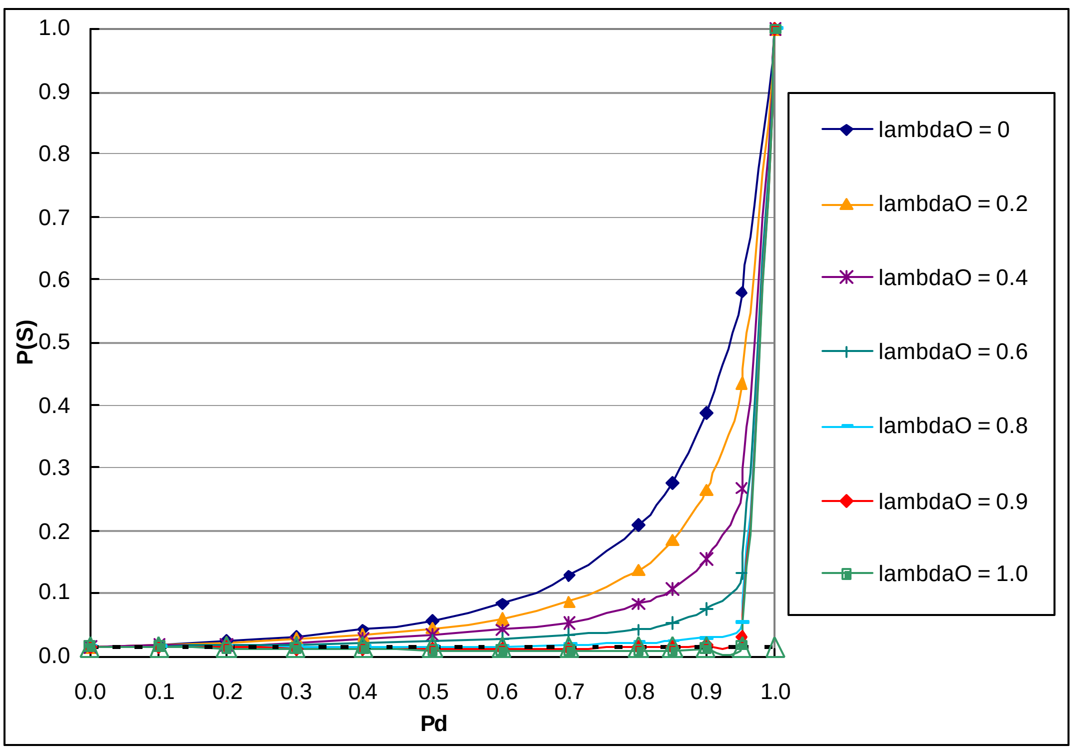

Figure 51. Probability of Safe Minefield Transit, *L=6, w=1.0,* l_M *=0.7*.

All the values located below the dotted line in the above graphs are the probabilities of a safe minefield transit which are less than or equal to that of the no sensor case. The tables and graphs clearly indicate a pattern. The plots of the data above show that a relationship exists between the probability of a safe minefield transit and the rate of occurrence of NOMBOs in the minefield. In addition, it is possible to determine that, as the rate of occurrence of mines increases, the region of the rate of occurrence of NOMBOs that result in the probability of a safe minefield transit when using a sensor falling below that of the no sensor case also increases.

3. Mine and False Alarm Case

The tables below display the probabilities of a safe minefield transit, when the rate of occurrence of NOMBOs is 0.0 and the rate of occurrence of mines is 0.2, 0.6, and 1.0, respectively. In addition, the rate of occurrence of false alarms is a function of the probability of detection and the detection index as described in Appendix A. The detection index ranges from 0.0 to 10.0. This shows the effect of occurrence of false alarms on the probability of a safe minefield transit as a function of the rate of occurrence of mines in the field.

Detection Index (*d*)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	0.186	0.210	0.238	0.275	0.311	0.378	0.437	0.516	0.645	1.000
0.2	0.165	0.188	0.215	0.246	0.285	0.335	0.403	0.466	0.573	0.698	1.000
0.4	0.165	0.188	0.216	0.248	0.289	0.340	0.409	0.477	0.573	0.716	1.000
0.6	0.165	0.189	0.217	0.250	0.292	0.340	0.409	0.486	0.586	0.732	1.000
0.8	0.165	0.189	0.217	0.252	0.292	0.344	0.413	0.486	0.596	0.744	1.000
1.0	0.165	0.189	0.218	0.252	0.294	0.344	0.413	0.492	0.596	0.744	1.000
2.0	0.165	0.189	0.219	0.254	0.297	0.351	0.420	0.502	0.611	0.761	1.000
4.0	0.165	0.190	0.219	0.255	0.299	0.353	0.423	0.510	0.623	0.775	1.000
6.0	0.165	0.190	0.219	0.255	0.299	0.354	0.424	0.512	0.628	0.780	1.000
8.0	0.165	0.190	0.219	0.255	0.299	0.354	0.424	0.513	0.628	0.783	1.000
10.0	0.165	0.190	0.219	0.255	0.299	0.354	0.424	0.513	0.629	0.785	1.000

Table 36. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.3$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.058	0.069	0.082	0.101	0.121	0.163	0.201	0.256	0.364	1.000
0.2	0.050	0.059	0.072	0.087	0.108	0.137	0.184	0.228	0.320	0.441	1.000
0.4	0.050	0.060	0.072	0.088	0.111	0.141	0.188	0.238	0.320	0.471	1.000
0.6	0.050	0.060	0.073	0.089	0.112	0.141	0.188	0.247	0.335	0.497	1.000
0.8	0.050	0.060	0.073	0.090	0.112	0.144	0.192	0.247	0.349	0.518	1.000
1.0	0.050	0.0.60	0.073	0.090	0.114	0.144	0.192	0.254	0.349	0.518	1.000
2.0	0.050	0.060	0.074	0.091	0.116	0.149	0.199	0.265	0.368	0.550	1.000
4.0	0.050	0.060	0.074	0.092	0.117	0.151	0.201	0.273	0.385	0.578	1.000
6.0	0.050	0.060	0.074	0.092	0.117	0.152	0.202	0.275	0.392	0.588	1.000
8.0	0.050	0.060	0.074	0.092	0.117	0.152	0.202	0.276	0.393	0.593	1.000
10.0	0.050	0.060	0.074	0.092	0.117	0.152	0.202	0.276	0.394	0.596	1.000

Table 37. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.5$

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.015	0.018	0.022	0.027	0.034	0.042	0.061	0.077	0.102	0.154	1.000
0.2	0.015	0.018	0.023	0.029	0.037	0.050	0.072	0.093	0.145	0.214	1.000
0.4	0.015	0.018	0.023	0.029	0.039	0.052	0.075	0.100	0.145	0.242	1.000
0.6	0.015	0.019	0.023	0.030	0.039	0.052	0.075	0.106	0.157	0.267	1.000
0.8	0.015	0.019	0.023	0.030	0.039	0.054	0.077	0.106	0.167	0.290	1.000
1.0	0.015	0.019	0.024	0.030	0.040	0.054	0.077	0.111	0.167	0.290	1.000
2.0	0.015	0.019	0.024	0.031	0.041	0.056	0.081	0.118	0.183	0.326	1.000
4.0	0.015	0.019	0.024	0.031	0.042	0.057	0.083	0.124	0.198	0.360	1.000
6.0	0.015	0.019	0.024	0.031	0.042	0.058	0.083	0.125	0.204	0.373	1.000
8.0	0.015	0.019	0.024	0.031	0.042	0.058	0.083	0.126	0.205	0.380	1.000
10.0	0.015	0.019	0.024	0.031	0.042	0.058	0.083	0.126	0.206	0.384	1.000

Table 38. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.7$.

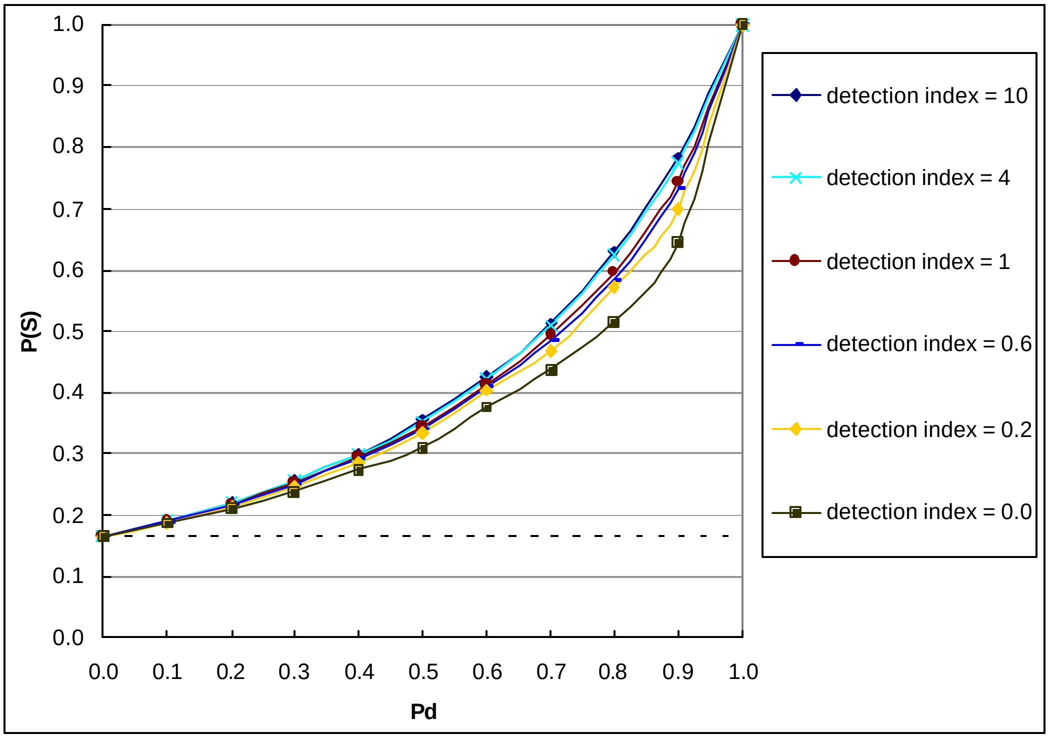

Figure 52. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.3$.

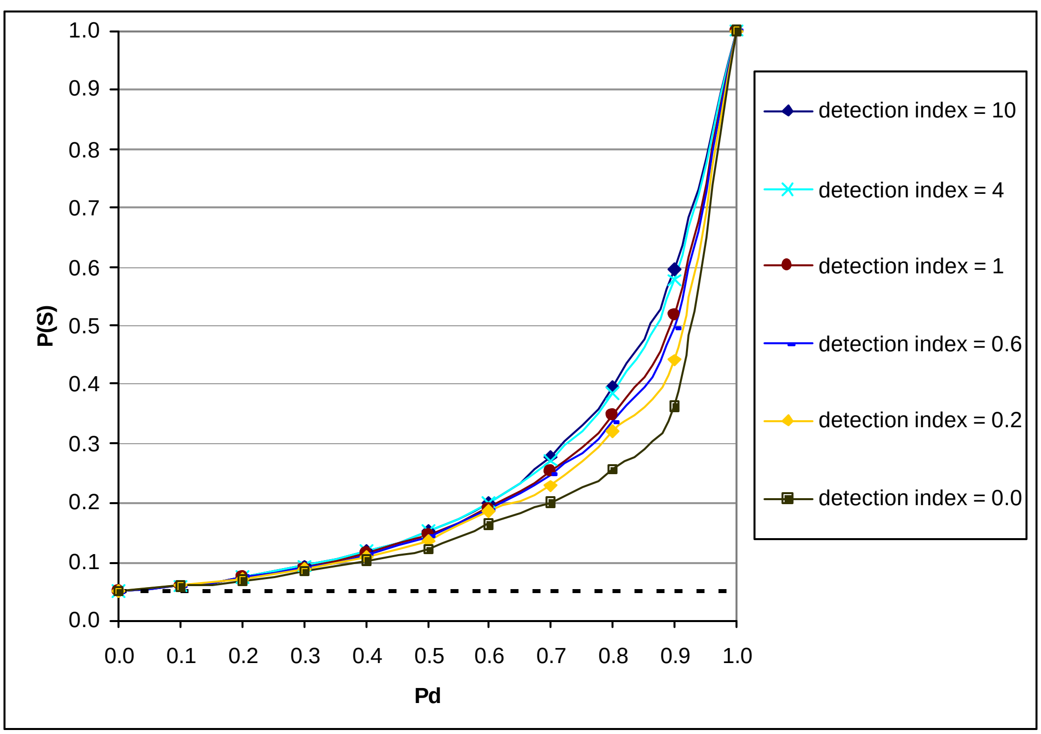

Figure 53. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.5$.

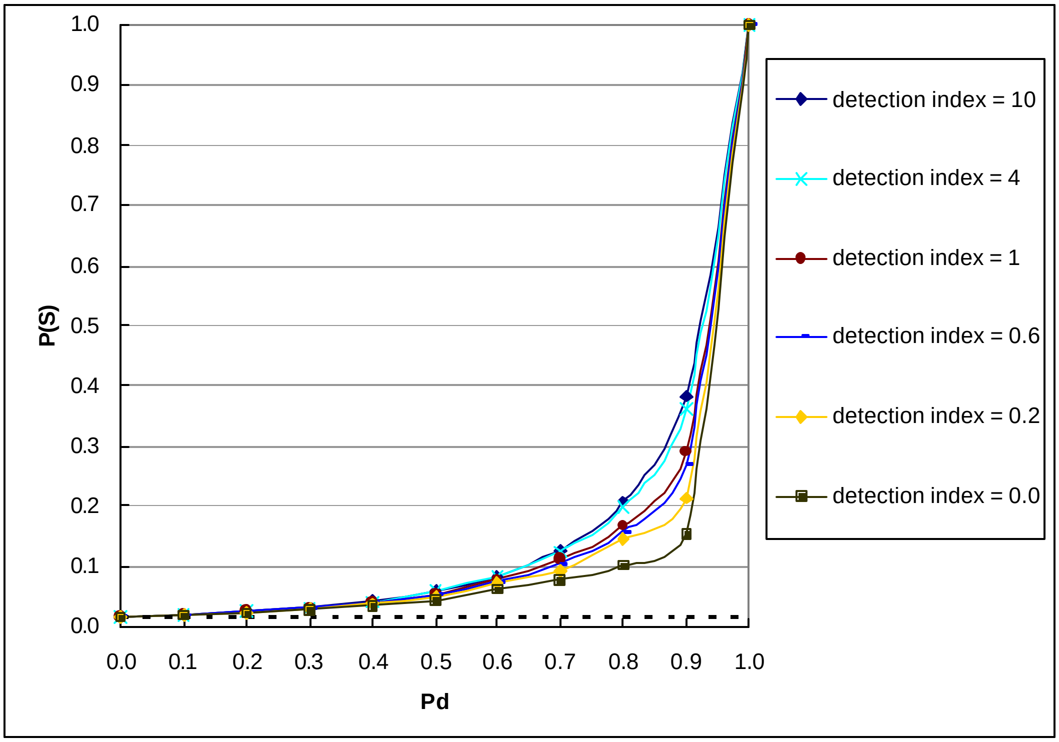

Figure 54. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.0$, $l_M=0.7$.

The tables and graphs clearly show the effect of false alarms on the probability of a safe minefield transit. As the detection index increases and the rate of occurrence of false alarms decreases, the probability of safe minefield transit increases. For a fixed probability of detection, the probability of a safe minefield transit is essentially constant for detection indices greater than 4. The probability of a safe minefield transit with a sensor is always greater than the probability of a safe minefield transit in a no sensor case.

4. Mine, NOMBO, and False Alarm Case

The tables below display the probabilities of a safe minefield transit when the rate of the occurrence of NOMBOs is 0.6 and the rate of occurrence of mines is 0.3, 0.5, and 0.7 respectively, and the rate of occurrence of false alarms is a function of the probability of detection and the detection index as described in Appendix A. These results show the effect of NOMBOs and false alarms on the probability of a safe minefield transit as a function of the rate of occurrence of mines in the field.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.165	**0.169**	0.175	0.179	0.187	0.185	0.191	0.201	0.193	0.202	1.000
0.2	0.165	0.172	0.180	0.187	0.199	0.212	0.218	0.236	0.261	0.287	1.000
0.4	0.165	0.172	0.181	0.190	0.203	0.217	0.228	0.251	0.271	0.325	1.000
0.6	0.165	0.173	0.182	0.192	0.206	0.217	0.236	0.263	0.291	0.361	1.000
0.8	0.165	0.173	0.182	0.194	0.206	0.222	0.243	0.263	0.309	0.391	1.000
1.0	0.165	0.173	0.183	0.194	0.208	0.222	0.243	0.273	0.309	0.391	1.000
2.0	0.165	0.173	0.183	0.196	0.211	0.230	0.253	0.288	0.336	0.440	1.000
4.0	0.165	0.173	0.184	0.197	0.213	0.233	0.260	0.299	0.360	0.485	1.000
6.0	0.165	0.173	0.184	0.197	0.213	0.234	0.262	0.303	0.370	0.502	1.000
8.0	0.165	0.173	0.184	0.197	0.213	0.234	0.263	0.304	0.372	0.511	1.000
10.0	0.165	0.173	0.184	0.197	0.213	0.234	0.263	0.305	0.374	0.515	1.000

Table 39. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.3$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	**0.051**	**0.052**	**0.053**	**0.055**	**0.053**	**0.054**	**0.055**	**0.050**	**0.049**	1.000
0.2	0.050	**0.052**	**0.054**	0.056	0.060	0.065	0.065	0.071	0.074	0.077	1.000
0.4	0.050	**0.052**	**0.055**	0.058	0.062	0.067	0.070	0.078	0.082	0.099	1.000
0.6	0.050	**0.052**	0.055	0.059	0.064	0.067	0.074	0.084	0.092	0.117	1.000
0.8	0.050	**0.052**	0.055	0.059	0.064	0.070	0.077	0.084	0.101	0.134	1.000
1.0	0.050	**0.052**	0.056	0.059	0.065	0.070	0.077	0.089	0.101	0.134	1.000
2.0	0.050	**0.052**	0.056	0.060	0.066	0.074	0.082	0.097	0.116	0.165	1.000
4.0	0.050	**0.052**	0.056	0.061	0.067	0.075	0.086	0.103	0.131	0.199	1.000
6.0	0.050	**0.052**	0.056	0.061	0.067	0.076	0.087	0.105	0.137	0.212	1.000
8.0	0.050	**0.052**	0.056	0.061	0.067	0.076	0.087	0.106	0.138	0.220	1.000
10.0	0.050	**0.052**	0.056	0.061	0.067	0.076	0.088	0.106	0.139	0.224	1.000

Table 40. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.5$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.015	**0.015**	**0.015**	**0.015**	**0.015**	**0.014**	**0.014**	**0.014**	**0.012**	**0.011**	1.000
0.2	0.015	**0.015**	**0.016**	**0.016**	0.017	0.018	0.018	0.019	0.022	0.020	1.000
0.4	0.015	**0.015**	**0.016**	**0.017**	0.018	0.019	0.020	0.021	0.022	0.026	1.000
0.6	0.015	**0.015**	**0.016**	**0.017**	0.018	0.019	0.021	0.024	0.025	0.031	1.000
0.8	0.015	**0.015**	**0.016**	**0.017**	0.018	0.020	0.022	0.024	0.028	0.037	1.000
1.0	0.015	**0.016**	**0.016**	**0.017**	0.019	0.020	0.022	0.025	0.028	0.037	1.000
2.0	0.015	**0.016**	**0.017**	**0.018**	0.019	0.022	0.024	0.028	0.034	0.049	1.000
4.0	0.015	**0.016**	**0.017**	0.018	0.020	0.022	0.025	0.031	0.039	0.062	1.000
6.0	0.015	**0.016**	**0.017**	0.018	0.020	0.022	0.026	0.031	0.042	0.068	1.000
8.0	0.015	**0.016**	**0.017**	0.018	0.020	0.022	0.026	0.032	0.042	0.072	1.000
10.0	0.015	**0.016**	**0.017**	0.018	0.020	0.022	0.026	0.032	0.043	0.074	1.000

Table 41. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.7$.

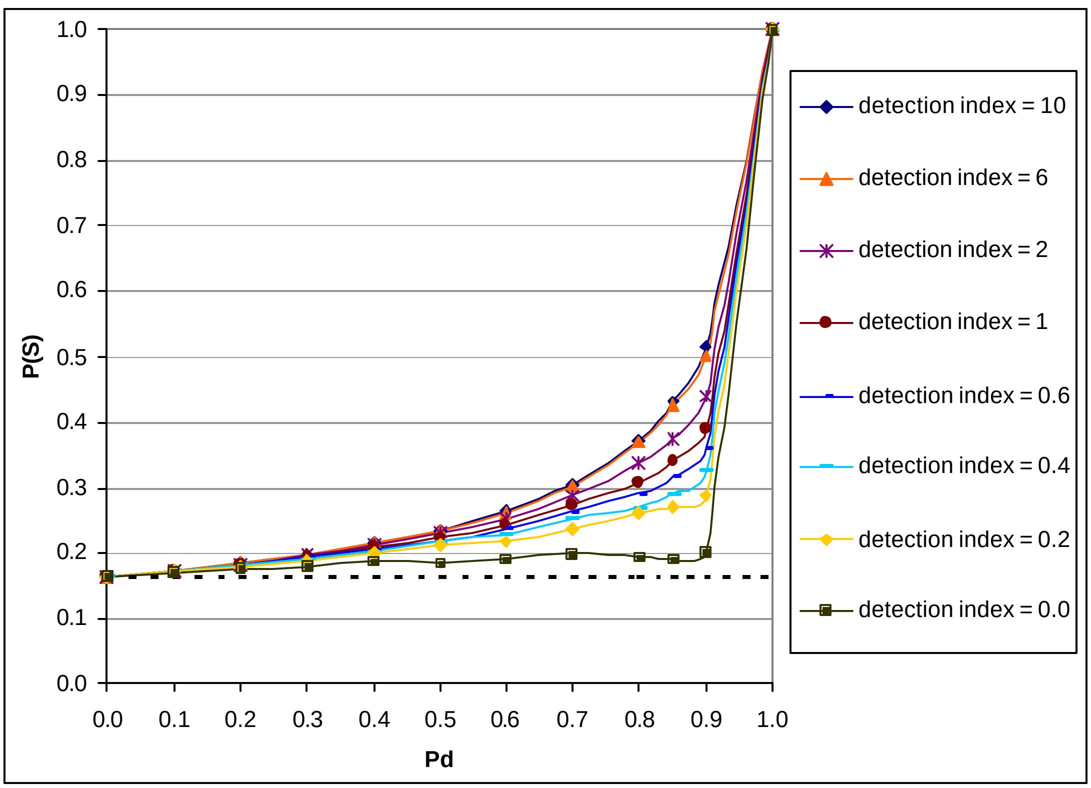

Figure 55. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.3$.

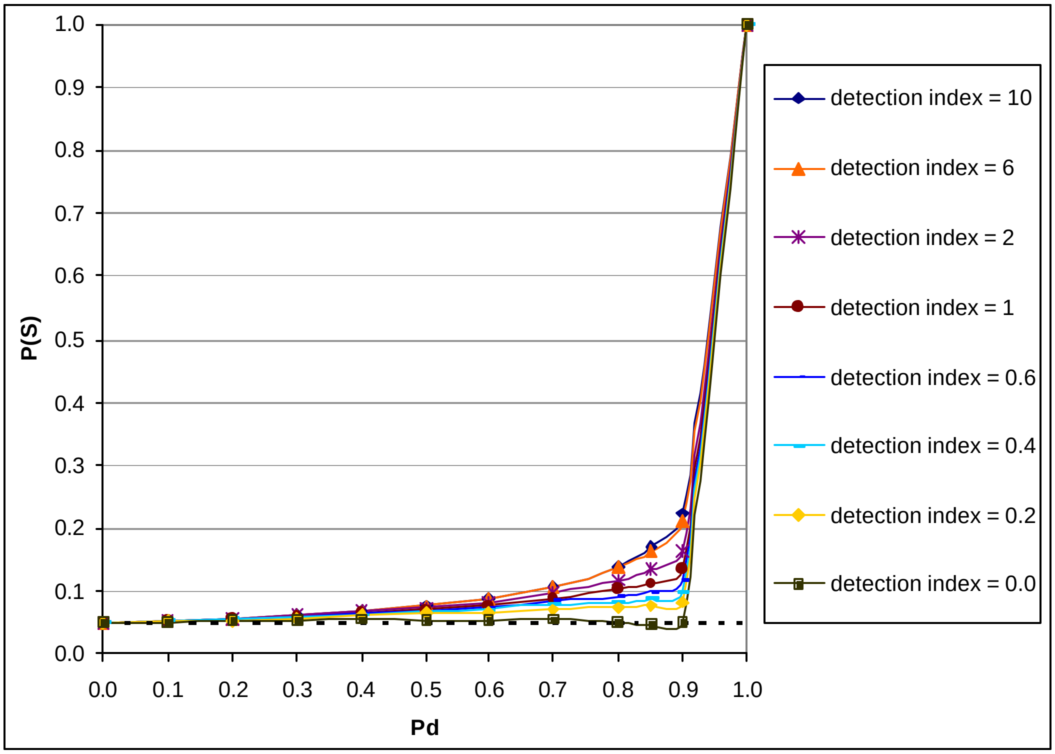

Figure 56. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.5$.

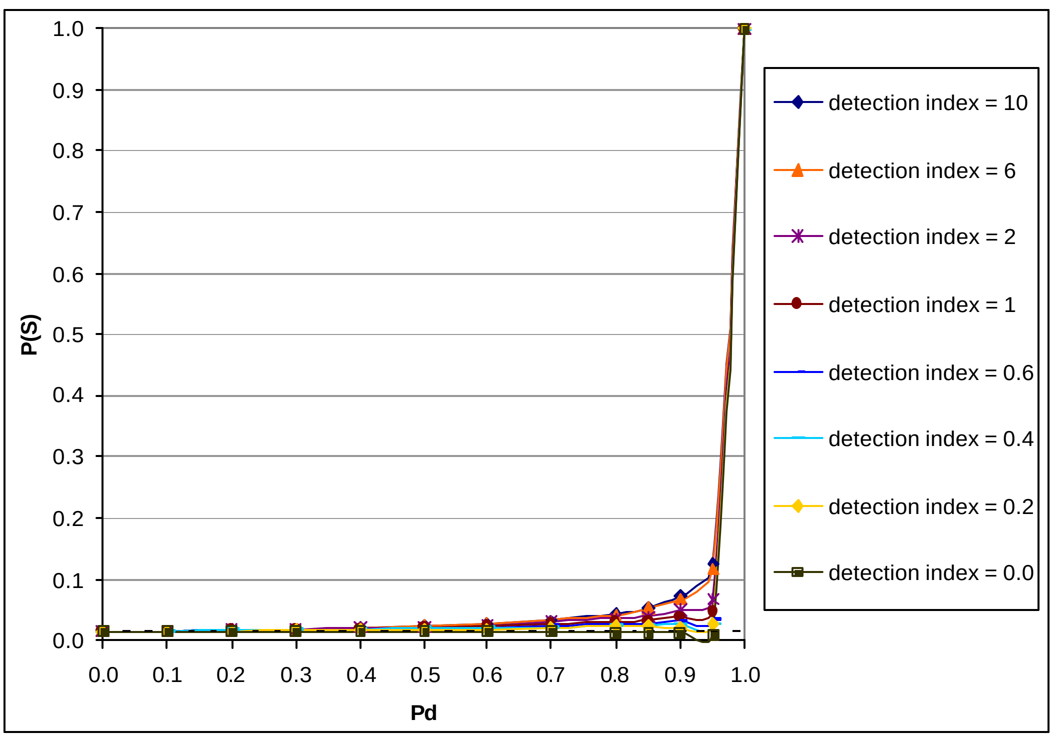

Figure 57. Probability of Safe Minefield Transit, $L=6$, $w=1.0$, $l_O=0.6$, $l_M=0.7$.

The colored boxes in the above tables and all the values located below the dotted line in the above graphs indicate that the probability of a safe minefield transit with a sensor is less than or equal to that with no sensor. The tables above also show that, as the detection index increases, the range of sensor detection probabilities for which using a sensor results in a smaller probability of a safe minefield transit through the field than that for a no sensor case becomes smaller.

D. DISCUSSION

In comparing the four cases, it appears that the probability of a safe minefield transit is highly dependent on the rate of occurrence of mines (l_M) in the minefield. Detected NOMBOs and false alarms cause the ship to travel greater distances within the field. Thus, it is possible for use of the sensor to decrease the probability of successful transit of the minefield. However, if the probability of detection is large enough, then the advantage of being able to detect an encountered mine outweighs the disadvantage of a longer distance traveled. The results also show that, for a constant probability of detection, the probability of a safe minefield transit is nearly constant for detection indices greater than or equal to 4.

The MOAM model represents better avoidance maneuvers than that of the SMT model. However, results from the SMT model indicate two importance relationships. Increasing the rate of occurrence of NOMBOs and the false alarm rate decreases the probability of a successful transit. Increasing the rate of occurrence of NOMBOs and the false alarm rate increases the distance traveled by the successfully transiting ship in the minefield. This increased distance results in more time being spent attempting to cross the field. Thus, even if the ship successfully transits the field, it may take an unacceptable amount of time to do so. Hence, the successful employment of mine avoidance tactics without a sensor that can classify mines and NOMBOs may be limited to those situations for which the rate of occurrence of NOMBOs and false alarm rates are small. These conclusions are also suggested by the results of the more complicated MOAM model. The MOAM model results suggest that these conclusions will apply generally and are not artifacts of the model representation of the avoidance maneuvering.

In this study, the step of the classification of an object that is detected is not considered. Thus, when the ship detects something in the minefield, it just attempts to avoid the object detected without classification. However, the results of the mine only case and the mine + false alarm case can be used to study the advantage of having perfect classification capability for mines and NOMBOs. Because, when the ship has a sensor that classifies objects perfectly, even though NOMBOs exist in the minefield, the ship can classify those correctly and continue to proceed as if they were not there. Thus, the results for perfect classification are the same as those of the mine only case and the mine + false alarm case. For instance, in Figure 36, when the probability of detection is 0.7, the probability of safe minefield transit of mine only case is 0.513, whereas that of mine and NOMBO case is 0.422. Here, the mine only case can be considered as a perfect classification case, and the mine and NOMBO case can be considered as a no classification case. When the ship has a perfect classification sensor, the probability of safe minefield transit is higher than that with no classification sensor; perfect classification results in an approximate 22% increase in the probability of safe transit over having no capability for classification. It is also noted that if the probability of detection is increased to 0.8, then the probability of safe minefield transit in the mine and NOMBO case is increased to .526, which is roughly equivalent to the improvement possible by adding perfect classification capability. In this manner, the model can be used to quantify the benefits of alternative investments in either technology that would increase the probability of detection or technology that adds significant classification capability. These quantified benefits could then be used in a complete cost-benefit analysis.

VIII. COMPARISON BETWEEN THE SIMPLE AND THE MOAM MODEL

A. INTRODUCTION

As studied in the previous chapter, when only mines exist in the minefield or mines exist and false alarms occur, the results displayed for the SMT model and MOAM models indicate that it is always better to use the sensor. Thus, for the purpose of comparing the difference between the two models, the mine + NOMBO case and the mine + NOMBO + false alarm case are analyzed.

The purpose of this chapter is to examine what is gained by adding more modeling complexity to the minefield transit model by comparing the results obtained with the two models to see if they are significantly different. This is accomplished by varying the rate of the occurrence of NOMBOs (l_O) and the detection index (d) respectively, calculating the resulting probabilities of successful transit, and comparing results to those with the no sensor case when a ship transits the minefield along a direct, straight line without a sensor. If the probability of a safe minefield transit ($P(S)$) when using a sensor is less than or equal to that in a no sensor case exists, there is no benefit to the ship using a sensor to transit the minefield. The analytical models are used to get the results in this chapter.

B. INPUT PARAMETERS

The table below shows the input parameters used in the analytical models.

Environment	Minefield Distance (L)	Mine Actuation width of ship (w)	Rate of Mine (l_M)	Rate of NOMBO (l_0)	Detection Index (d)
Mine & NOMBO	6	.5	1.0	0.0 ~ 1.0	-
Mine, NOMBO & False Alarm	6	.5	1.0	.6	0.0 ~ 10.0

Table 42. Input Parameters for Each Environment.

C. PROBABILITY OF SAFE MINEFIELD TRANSIT

1. Mine and NOMBO Case

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.055	0.061	0.070	0.080	0.095	0.116	0.149	0.208	0.344	1.000
0.1	0.050	0.054	0.059	0.066	0.074	0.086	0.103	0.130	0.180	0.301	1.000
0.2	0.050	0.053	0.057	0.062	0.068	0.078	0.092	0.114	0.156	0.261	1.000
0.3	0.050	0.052	0.054	0.058	0.063	0.070	0.081	0.099	0.133	0.223	1.000
0.4	0.050	0.051	0.052	0.054	0.058	0.063	0.071	0.086	0.114	0.190	1.000
0.5	0.050	**0.050**	**0.050**	0.051	0.053	0.057	0.063	0.074	0.096	0.159	1.000
0.6	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.081	0.133	1.000
0.7	0.050	**0.048**	**0.046**	**0.045**	**0.045**	**0.046**	**0.048**	0.054	0.068	0.110	1.000
0.8	0.050	**0.047**	**0.044**	**0.042**	**0.041**	**0.041**	**0.042**	**0.046**	0.057	0.090	1.000
0.9	0.050	**0.046**	**0.042**	**0.039**	**0.038**	**0.037**	**0.037**	**0.040**	**0.047**	0.074	1.000
1.0	0.050	**0.045**	**0.040**	**0.037**	**0.034**	**0.033**	**0.032**	**0.034**	**0.039**	0.060	1.000

Table 43. Probability of Safe Minefield Transit by Using the SMT Model, *L=6, w=0.5,* $l_M = 1.0$.

lambdaO parameter	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.063	0.081	0.105	0.138	0.184	0.248	0.339	0.474	0.677	1.000
0.1	0.050	0.063	0.080	0.104	0.136	0.180	0.242	0.332	0.464	0.668	1.000
0.2	0.050	0.062	0.079	0.102	0.133	0.176	0.236	0.324	0.454	0.658	1.000
0.3	0.050	0.062	0.078	0.100	0.130	0.172	0.230	0.315	0.444	0.648	1.000
0.4	0.050	0.062	0.077	0.099	0.128	0.168	0.224	0.307	0.433	0.636	1.000
0.5	0.050	0.061	0.077	0.097	0.125	0.163	0.218	0.298	0.421	0.624	1.000
0.6	0.050	0.061	0.076	0.095	0.122	0.159	0.212	0.289	0.409	0.610	1.000
0.7	0.050	0.061	0.075	0.094	0.119	0.155	0.205	0.280	0.396	0.596	1.000
0.8	0.050	0.060	0.074	0.092	0.117	0.151	0.199	0.271	0.383	0.580	1.000
0.9	0.050	0.060	0.073	0.090	0.114	0.146	0.193	0.261	0.369	0.564	1.000
1.0	0.050	0.059	0.072	0.089	0.111	0.142	0.186	0.251	0.355	0.546	1.000

Table 44. Probability of Safe Minefield Transit by Using the MOAM Model, *L=6, w=0.5,* $l_M = 1.0$.

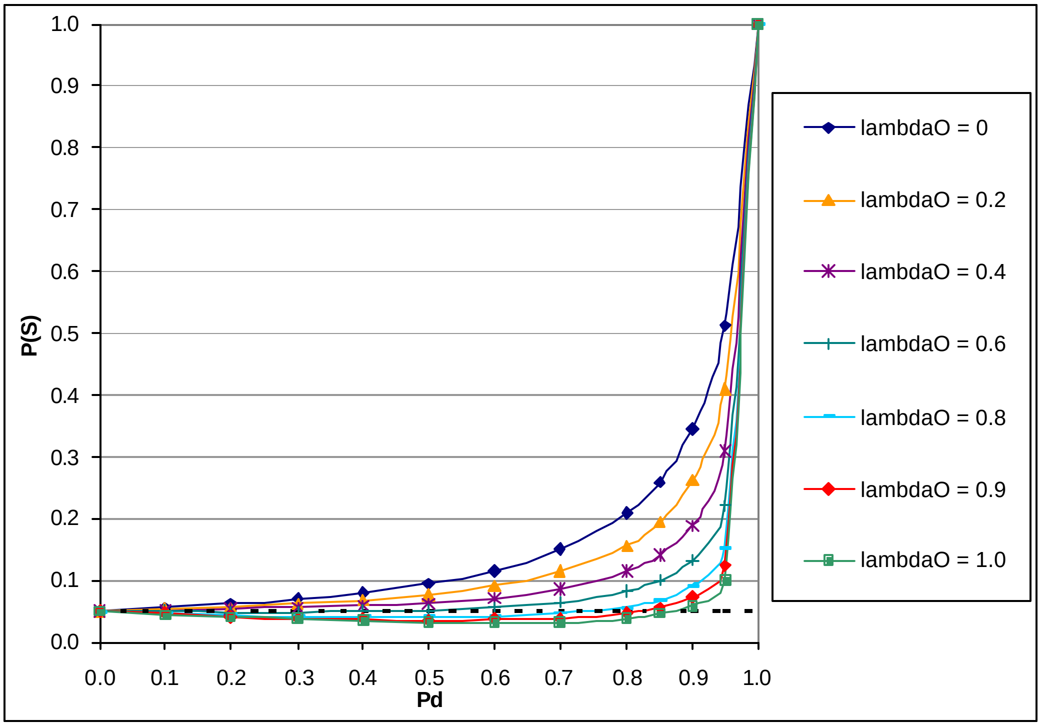

Figure 58. Probability of Safe Minefield Transit by Using the SMT Model, $L=6$, $w=0.5$, $l_M=1.0$.

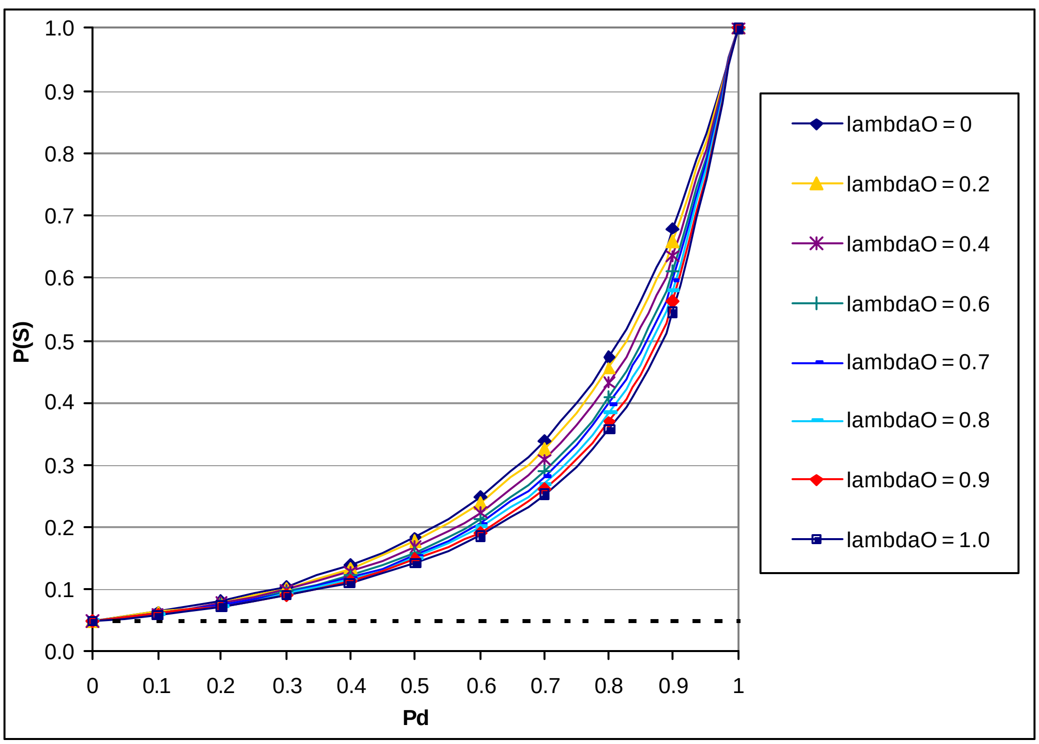

Figure 59. Probability of Safe Minefield Transit by Using the MOAM Model, $L=6$, $w=0.5$, $l_M=1.0$.

The tables and graphs above show the probability of safe minefield transit, when the rate of occurrence of mines is 1.0. The rate of occurrence of NOMBOs ranges from 0.0 to 1.0 in increments of 0.1. There are no false alarms. The colored boxes in the above table indicate that the probability of a safe minefield transit is less than or equal to that of a no sensor case (probability of detection = 0.0).

Comparison of the Tables and Graphs above shows that there is no case displayed for the MOAM model for which it is better not to use the sensor. However since the SMT model assumes that the ship always returns to the entry to the field, the SMT model has displayed cases in which it is better not to use the sensor. The results of the simpler SMT model are more pessimistic than those of the more complex MOAM model, which is not surprising. This suggests that even greater complexity in maneuver modeling may be desirable for some purposes.

2. Mine, NOMBO, and False Alarm Case

The tables below display the probabilities of a safe minefield transit when the rate of the occurrence of NOMBOs is 0.6 and the rate of occurrence of mines is 1.0. The rate of occurrence of false alarms is a function of the probability of detection and the detection index as described in Appendix A. The colored boxes in the below table indicate that the probability of a safe minefield transit is less than or equal to that of a no sensor case (probability of detection = 0.0).

In this case, the situation is more challenging for the ship than that of the previous section case, because not only is the rate of occurrence of mines high, but also false alarms can occur. As a result, the SMT analytical model (or simulation) has more cases for which the probability of a safe minefield transit with a sensor is less than or equal to that with no sensor than in the previous section. There is still no case displayed for the MOAM model for which it is better not to use the sensor. The results of the simpler SMT model are more pessimistic than those of the more complex MOAM model, which is not surprising. This suggests that even greater complexity in maneuver modeling may be desirable for some purposes.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	**0.045**	**0.041**	**0.037**	**0.034**	**0.030**	**0.028**	**0.026**	**0.024**	**0.023**	1.000
0.2	0.050	**0.047**	**0.045**	**0.042**	**0.040**	**0.038**	**0.037**	**0.036**	**0.038**	**0.041**	1.000
0.4	0.050	**0.048**	**0.046**	**0.044**	**0.042**	**0.041**	**0.040**	**0.041**	**0.042**	0.051	1.000
0.6	0.050	**0.048**	**0.046**	**0.045**	**0.044**	**0.043**	**0.043**	**0.044**	**0.047**	0.056	1.000
0.8	0.050	**0.048**	**0.047**	**0.045**	**0.044**	**0.044**	**0.044**	**0.046**	0.051	0.063	1.000
1.0	0.050	**0.048**	**0.047**	**0.046**	**0.045**	**0.045**	**0.046**	**0.048**	0.054	0.068	1.000
2.0	0.050	**0.049**	**0.048**	**0.047**	**0.047**	**0.048**	0.051	0.055	0.064	0.088	1.000
4.0	0.050	**0.049**	**0.048**	**0.048**	**0.048**	0.050	0.054	0.060	0.074	0.111	1.000
6.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.062	0.078	0.122	1.000
8.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.080	0.127	1.000
10.0	0.050	**0.049**	**0.048**	**0.048**	**0.049**	0.051	0.055	0.063	0.081	0.130	1.000

Table 45. Probability of Safe Minefield Transit by Using the SMT Model, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

Detection Index (d)	Probability of Detection										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.050	0.060	0.073	0.089	0.113	0.140	0.181	0.240	0.318	0.463	1.000
0.2	0.050	0.060	0.074	0.092	0.117	0.150	0.192	0.258	0.361	0.512	1.000
0.4	0.050	0.061	0.075	0.093	0.118	0.152	0.197	0.265	0.361	0.531	1.000
0.6	0.050	0.061	0.075	0.094	0.119	0.152	0.200	0.270	0.371	0.547	1.000
0.8	0.050	0.061	0.075	0.094	0.119	0.154	0.203	0.270	0.379	0.560	1.000
1.0	0.050	0.061	0.075	0.094	0.120	0.154	0.203	0.275	0.379	0.560	1.000
2.0	0.050	0.061	0.075	0.095	0.121	0.158	0.208	0.282	0.392	0.580	1.000
4.0	0.050	0.061	0.076	0.095	0.122	0.159	0.211	0.287	0.403	0.598	1.000
6.0	0.050	0.061	0.076	0.095	0.122	0.159	0.211	0.288	0.407	0.604	1.000
8.0	0.050	0.061	0.076	0.095	0.122	0.159	0.212	0.289	0.408	0.607	1.000
10.0	0.050	0.061	0.076	0.095	0.122	0.159	0.212	0.289	0.409	0.609	1.000

Table 46. Probability of Safe Minefield Transit by Using the MOAM Model, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

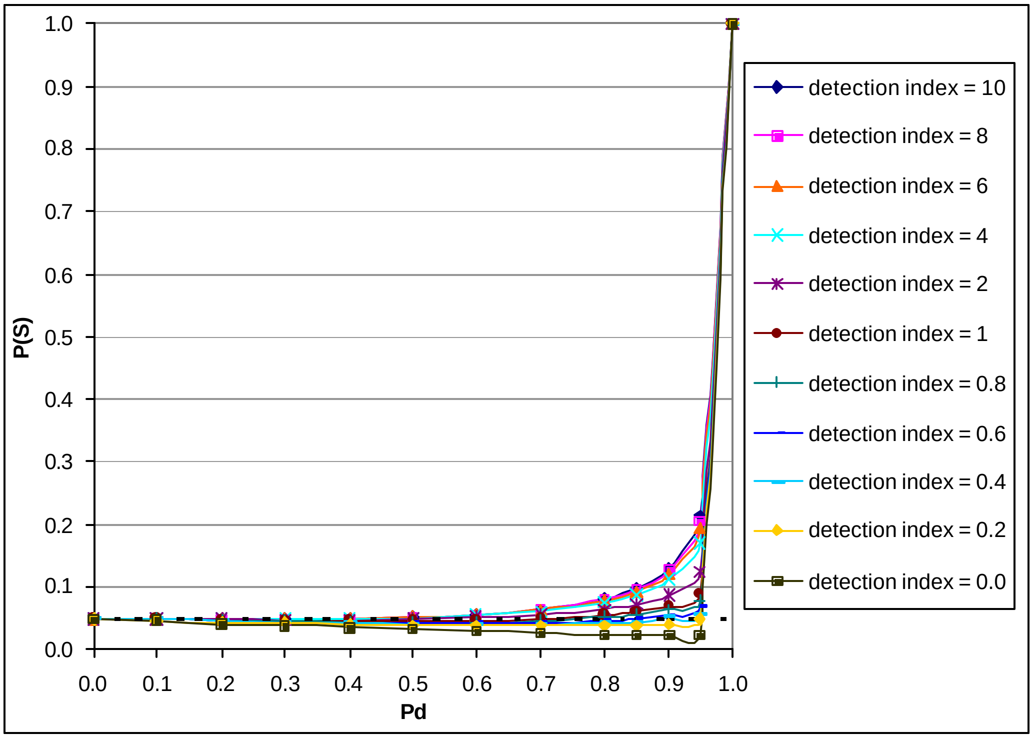

Figure 60. Probability of Safe Minefield Transit by Using the SMT Model, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

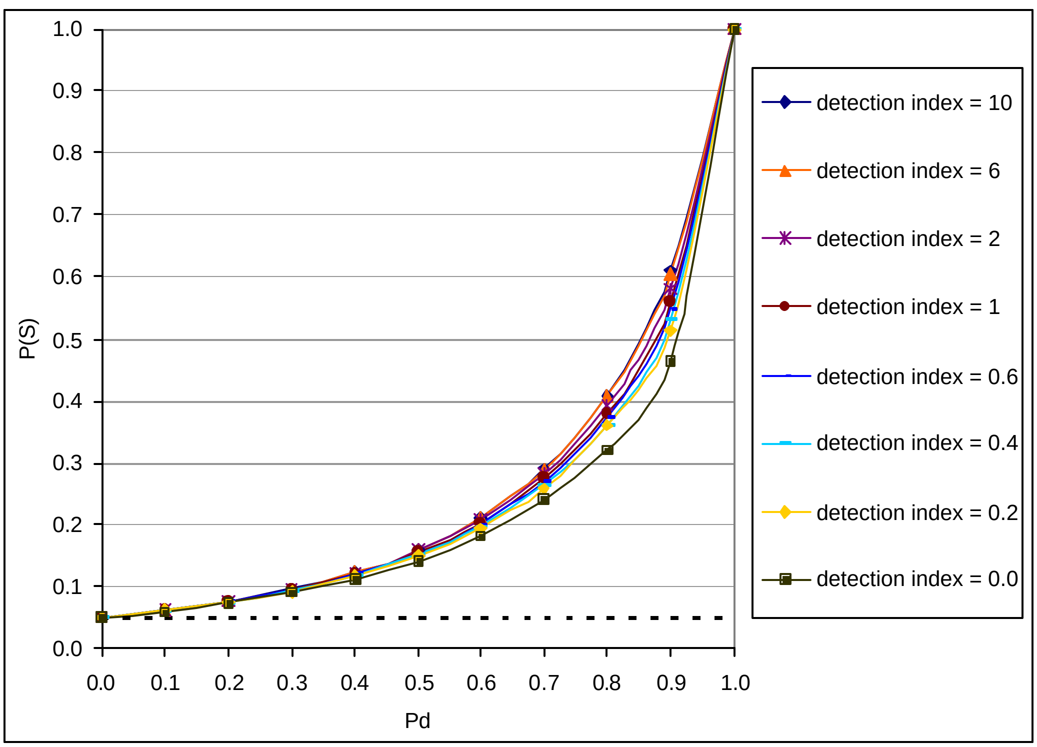

Figure 61. Probability of Safe Minefield Transit by Using the MOAM Model, $L=6$, $w=0.5$, $l_O=0.6$, $l_M=1.0$.

IX. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

In this thesis, two models of mine avoidance maneuvering are formulated. One is the Simple Minefield Transit (SMT) Model and the other is the Minefield Object Avoidance Maneuver (MOAM) Model.

In the Simple Minefield Transit (SMT) model, when the ship encounters a mine or NOMBO, or the sensor gives a false alarm, the ship retraces its route back to the entry to the minefield, moves to a different location and attempts to cross the field again along a straight path that does not intersect any of its previous attempts.

In the MOAM model, when the ship encounters a detected NOMBO or mine, or the sensor gives a false alarm, the ship attempts to go around the location of the detected object. The ship goes an avoiding distance to the right (for illustration, could alternatively go to the left). If the ship does not detect a NOMBO or mine, or the sensor does not give a false alarm and the ship survives the distance, then it once again proceeds to the end of the field. If the ship encounters a detected object or mine or the sensor gives a false alarm while going to the right, the ship backtracks and tries an avoiding distance to the left; if it does not detect an object and the sensor does not give a false alarm and survives during this avoidance path, it once again proceeds towards the end of the field. If the ship encounters a detected NOMBO or mine, or the sensor gives a false alarm in both directions and the ship survives, the ship goes back to the entry to the field and starts over again.

The results of this study demonstrate that if NOMBOs exist in the minefield, the probability of a safe minefield transit does not always increase with increasing sensor probability of detection, but sometimes decreases. That is, since detected NOMBOs and false alarms cause the ship to travel greater distances within the field, it is possible for use of the sensor to decrease the probability of successful transit of the minefield. However, if the probability of detection is high enough, then the advantage of being able to detect an encountered mine outweighs the disadvantage of a longer distance traveled. When there are no NOMBOs in the field, and the probability of detection increases, even

if false alarms occur and the rate of occurrence of false alarms is great, the probability of a safe minefield transit always increases. In other words, if it is possible to guarantee that no NOMBOs exist in the minefield, sensors must be used to transit the minefield, even though the detection index is low, because the probability of a safe minefield with a sensor is always greater than that with no sensor. However, in the real world, this situation seldom occurs. Thus, how can the ship transit the minefield safely? First, the rate of occurrence of unknown NOMBOs in the minefield should be reduced. The rate of occurrence of mines is not controllable since enemy forces deploy mines. However, surveying the bottom continuously during peacetime and keeping data about location of objects on the bottom can reduce the rate of occurrence of unidentified NOMBOs. Next, reducing the rate of false alarms can be accomplished by improving the sensor signal-to-noise ratio. Comparison of the results for which there are only mines in the field with those in which there are also NOMBOs suggest that the additional ability to be able to classify NOMBOs is important to successful employment of a mine avoidance tactic.

The histograms of distances traveled suggest that even if the ship successfully transits the minefield, it may need to transit a substantial distance while doing so. The distance traveled is a function of the rate of occurrence of NOMBOs and the rate of occurrence of false alarms. Thus, even if a ship transits the field successfully, it may not do so within an acceptable amount of time.

Successful use of the mine avoidance tactic without a sensor that can accurately classify mines and NOMBOs may be limited to those situations for which the rate of occurrence of NOMBOs and false alarm rates are small. Since similar conclusions are obtained from both models, the results suggest that these conclusions usually apply and are not artifacts of the model representation of avoidance maneuvering.

B. RECOMMENDED FOLLOW-ON RESEARCH

This thesis can be used as a basis for the study of extended and enhanced models and minefield transit tactics. In this thesis, the capability to classify objects that is detected is skipped. Thus, when the ship detects something in the minefield, it must return to the entry to the field or attempt to avoid the object detected without classification. Object classification will add time to the time to transit the field.

Additional research can study the tradeoffs of being able to classify objects with error and the ability to transit the field safely in a timely manner. Also the effect of being able to classify objects by varying the probabilities of detection for the mines and the NOMBOs can be studied.[10]

The models used in this thesis do not consider a speed of the ship and the resulting time to transit the field. Only the distance traveled is analyzed as the measure of effectiveness. Models can be formulated and studied that include the speed of the ship.

The MOAM model could be enhanced so that rather than the ship always returning to the entry point whenever it detects something in both directions, more complex paths involving partial retracings are tried. Simulation could be used to explore more complicated avoiding tactics, other distributions of mines and NOMBOs, more complicated mine actuation functions, and to assess the efficacy of the tactics in crossing a finite-width (rather than infinite width) minefield.

[10] In this thesis, the probability of detection of mine and that of NOMBO are considered as the same.

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APPENDIX A. ROC CURVE MODEL[11]

A. INTRODUCTION

A signal received by a sensor is not always easy to classify. Thus, it is important to keep in mind that the signal must be pulled out of an underlying blanket of noise as shown in Figure 62 below. Some of the thermal noise power created in the receiver will be amplified along with the incoming signal and may be the dominant form of noise to contend with where other interference, such as clutter or active jamming, is not an issue. Figure 62 shows that the receiver operator can increase the probability of detection by lowering the threshold for the minimum detectable signal. However, lowering the threshold increases the chances of a noise spike being large enough to mislead the radar to indicate that a target has been detected, when in fact, it was only noise in the receiver.

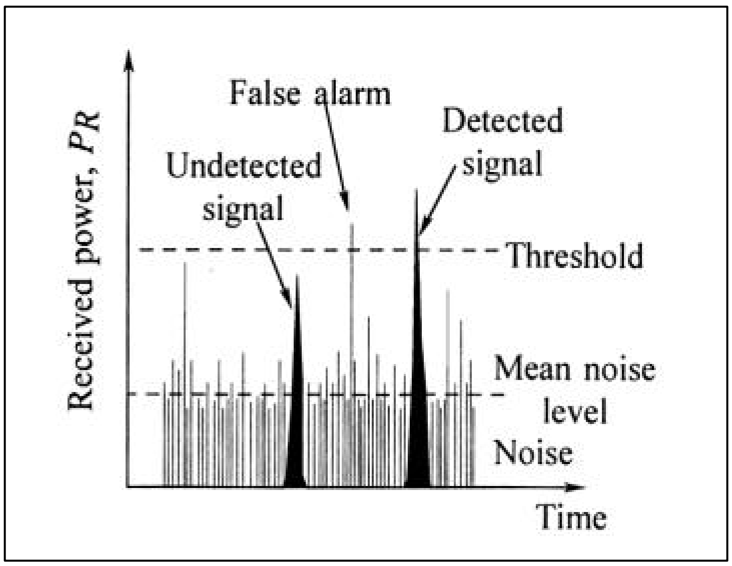

Figure 62. Sensor Signal Threshold.

As a result of these factors, detection is based on probability functions called the probability of detection (P_d) and the probability of false alarm (P_f), which are dependent on each other and the signal to noise ratio (S/N). Since P_d and P_f are always mutually interdependent, it is always necessary to specify them together in order to give complete meaning to either.

[11] A model that computes the probability of a false alarm (P_f) based on a given probability of detecting a mine or NOMBO (P_d) is presented in this section (Pilnick, 2002).

B. PROBABILITY OF DETECTION AND FALSE ALARM

Assume the probability of detection is P_d, when something is detected, given that a target is present, and the probability of a false alarm is P_f, when something is detected, given that a target is absent.

$$
\begin{aligned}
P_d &\equiv P\{call\text{"detect"} \mid \text{target present}\} \\
&= \text{detection probability} \\
P_f &\equiv P\{call\text{"detect"} \mid \text{target absent}\} \\
&= \text{false alarm probability}
\end{aligned}
$$

The table below shows the probability according to the threshold of a sensor.

Sensor reading	Given that target present	Given that target absent
Above threshold ➔ Call “detect”	Detection P_d	False alarm P_f
Below threshold ➔ Don’t call “detect”	Miss or fail to detect 1- P_d	--------- 1-P_f

Table 47. Probability According to the Threshold of a Sensor.

For many sensors, there is a user-selectable tradeoff between P_d and P_f. P_d can be made as close to 1 as desired, if an accompanying large P_f is acceptable. On the other hand, P_f can be made as close to 0 as desired, if an accompanying small P_d is acceptable.

C. ROC MODEL

A Receiver Operating Characteristic (ROC) curve is a plot of (P_d, P_f) pairs for a particular sensor. Assume that a single independent signal measurement is made. Let s be the known signal (voltage level) due to the target and let N be the random (Gaussian) electrical noise in the receiver (a random variable),

N ~ Normal dist(m , s^2)

Let V be the voltage level present at the receiver,

$V = N$, when no target is present V ~ Normal dist(m , s^2)

$V = s + N$, when a target is present V ~ Normal dist($m + s$, s^2)

Let v be the threshold level, and detection is called if and only if $V \geq v$.

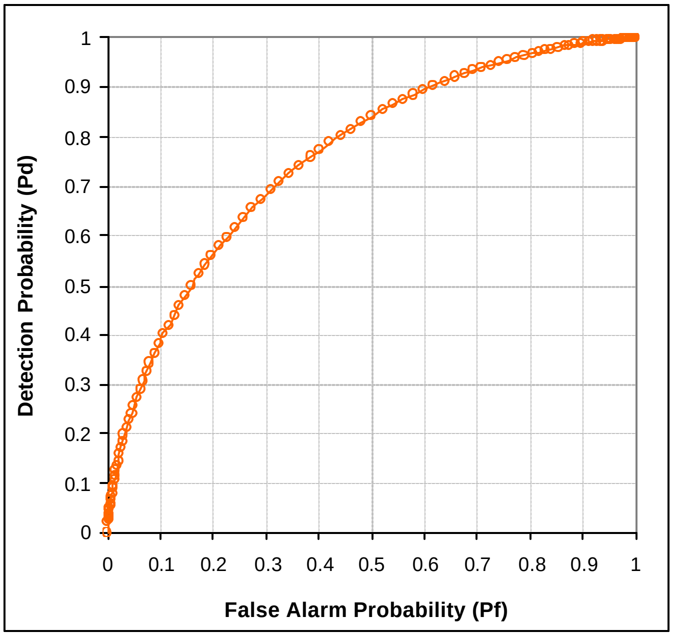

Figure 63. ROC Curve.

Figure 64 below describes the probability distribution for the voltage level at the receiver, V.

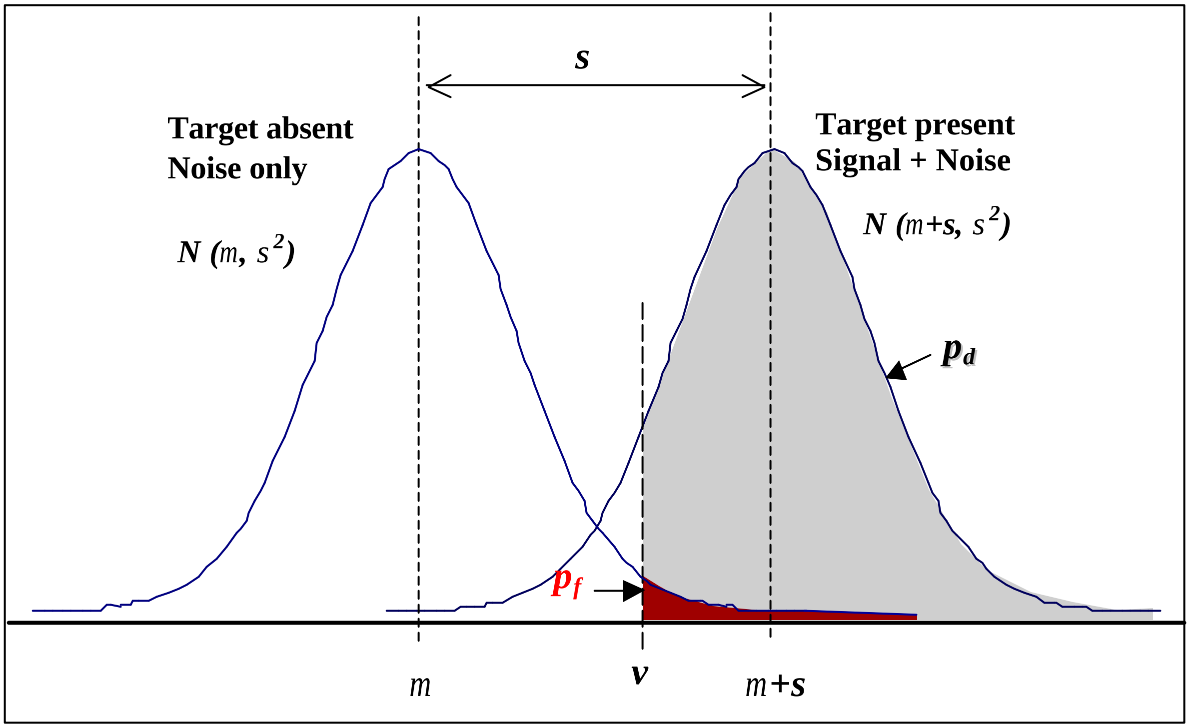

Figure 64. Probability Distribution for Voltage Level at the Receiver, V.

$$P_f \equiv P\{call\text{"detect"} \mid \text{target absent}\}$$
$$= P\{V \geq v \mid V \sim Normal(m, s^2)\}$$
$$P_f = 1 - \Phi\left(\frac{v-m}{s}\right)$$
$$P_d \equiv P\{call\text{"detect"} \mid \text{target present}\}$$
$$= P\{V \geq v \mid V \sim Normal(m+s, s^2)\}$$
$$P_d = 1 - \Phi\left(\frac{v-m-s}{s}\right)$$

The above equation can be written as

$$P_f = 1 - \Phi(x)$$
$$P_d = 1 - \Phi\left(x - \sqrt{d}\right)$$

where $x = \dfrac{v-m}{s}$, known as a normalized threshold

and $d = \dfrac{s^2}{s^2}$, known as a detection index.

The detection index is a dimensionless measure of the separation of the two density functions. For any specified *d*, a ROC curve can be generated by varying the dimensionless, normalized threshold x.

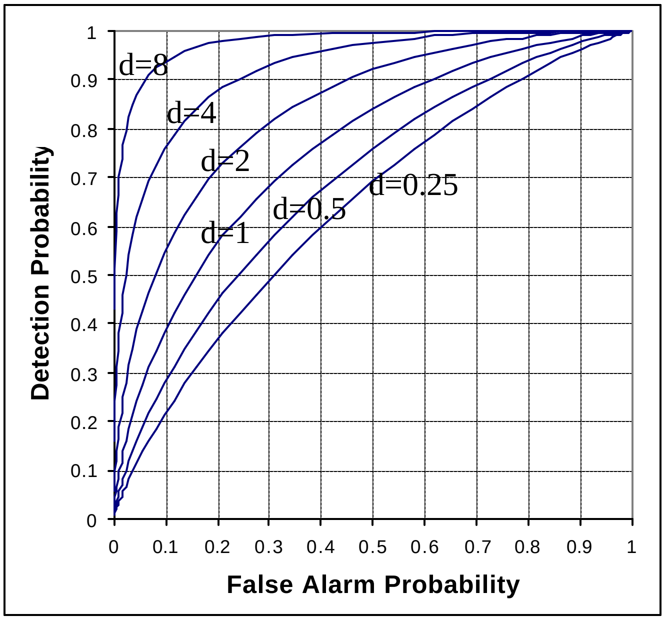

Figure 65. ROC curves for Various Detection Index (*d*).

Note that s^2 is proportional to the radar power generated and s^2 is proportional to noise power. Thus, the detection index is proportional to the signal-to-noise ratio (SNR) at the receiver. Doubling the SNR also doubles the detection index. The detection index allows the construction of a reasonable set of ROC curves based on the single parameter *d*.

Now, assume that *n* independent signal measurements are made and averaged. Let V_n be the average voltage level present at the receiver. As before, let *v* be the threshold level, and detection is called if and only if $V_n \geq v$.

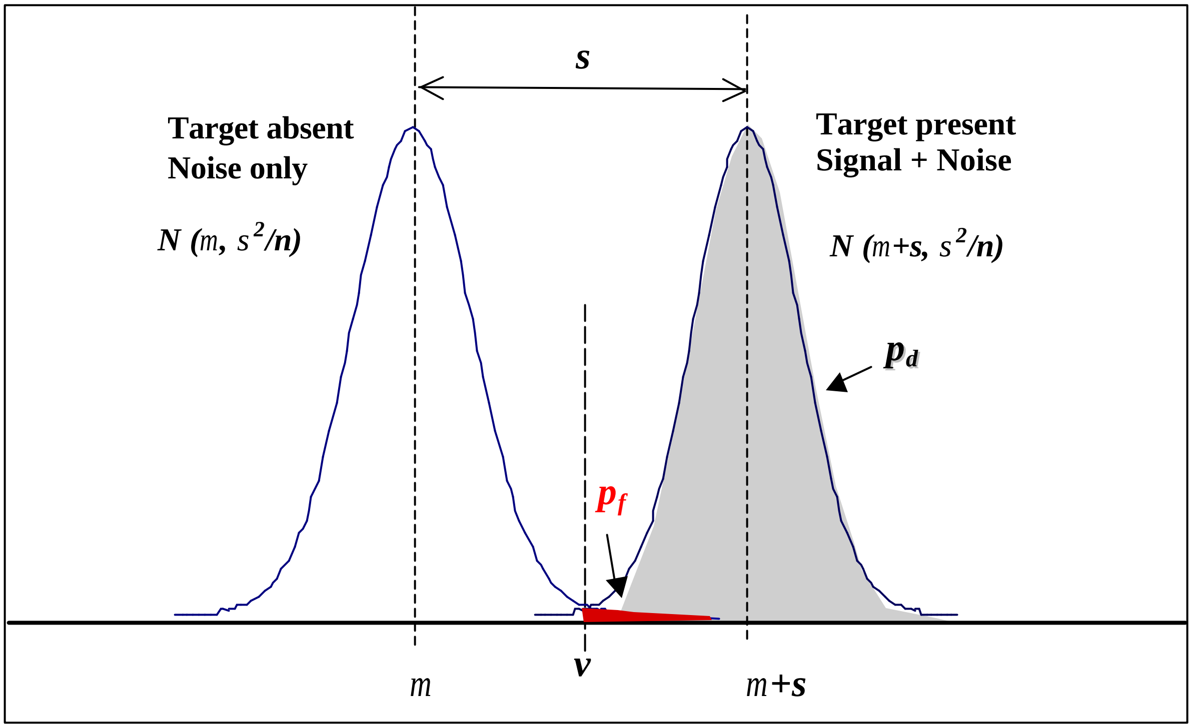

Figure 66. Probability Distribution for Voltage Level at the Receiver, V_n.

Now, if a target is absent,

$$V_n = \frac{1}{n}\sum_{i=1}^{n} N_i \ , \text{ where } N_i \sim Normal(m,\ s^2)$$
$$\text{so } V_n \sim Normal(m,\ s^2/n)$$
$$\text{and } P_f = P(V_n \geq v) = 1 - \Phi\left(\frac{v-m}{s/\sqrt{n}}\right)$$

If a target is present,

$$V_n = \frac{1}{n}\sum_{i=1}^{n} (N_i + s) \ , \text{ where } N_i \sim Normal(m,\ s^2)$$
$$\text{so } V_n \sim Normal(m+s,\ s^2/n)$$
$$\text{and } P_d = P(V_n \geq v) = 1 - \Phi\left(\frac{v-m-s}{s/\sqrt{n}}\right)$$

Now, the detection index is

$$d_n = \frac{s^2}{s^2/n} = n\left(\frac{s^2}{s^2}\right)$$

Therefore, averaging n independent signal measurements, i.e., "processing" the signal, effectively increases the SNR by a factor of n.

D. RATE OF OCCURRENCE OF FALSE ALARMS

Let l_F be the rate of occurrence of false alarms,

$$P_f = 1 - e^{-l_F|A|} = 1 - e^{-l_F(L*w)}$$

$$A = \text{rectangle with length } L \text{ and width } w$$

$$|A| = \text{area of } A$$

$$e^{-l_F(L*w)} = 1 - P_f$$

$$-l_F(L*w) = \ln(1-P_f)$$

$$l_F = -\frac{\ln(1-P_f)}{L*w}$$

As P_f can be derived from the ROC curve, l_F can be easily obtained.

Detection Index (d)	Probability of Detection (P_d)										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.0	0.000	0.081	0.159	0.274	0.345	0.500	0.579	0.655	0.788	0.885	0.999
0.2	0.000	0.036	0.081	0.159	0.212	0.274	0.421	0.500	0.579	0.788	0.999
0.4	0.000	0.023	0.055	0.115	0.159	0.212	0.345	0.421	0.579	0.726	0.999
0.6	0.000	0.014	0.036	0.081	0.115	0.212	0.274	0.345	0.500	0.655	0.999
0.8	0.000	0.014	0.036	0.055	0.115	0.159	0.212	0.345	0.421	0.579	0.999
1.0	0.000	0.008	0.023	0.055	0.081	0.159	0.212	0.274	0.421	0.579	0.999
2.0	0.000	0.003	0.008	0.023	0.036	0.055	0.115	0.159	0.274	0.421	0.999
4.0	0.000	0.000	0.001	0.005	0.008	0.023	0.036	0.055	0.115	0.212	0.999
6.0	0.000	0.000	0.000	0.001	0.003	0.005	0.014	0.023	0.036	0.115	0.999
8.0	0.000	0.000	0.000	0.000	0.000	0.001	0.005	0.008	0.023	0.055	0.999
10.0	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.003	0.008	0.023	0.999

Table 48. Probability of False Alarm by Using ROC Curve.

Table 48 above displays the probabilities of a false alarm that are used in the thesis as a function of the detection index and the probability of detection. Other possibilities exist.

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